J. Cent. South Univ. (2015) 22: 2809−2815
DOI: 10.1007/s11771-015-2812-3

Springer

Wavelet basis construction method based on separation blast vibration signal

LING Tong-hua(凌同华)[1], ZHANG Sheng(张胜)[1], CHEN Qian-qian(陈倩倩)[2], LI Jie(李洁)[3]

1. School of Civil Engineering and Architecture, Changsha University of Science & Technology, Changsha 410114, China;
2. Institute of Civil Engineering, Hunan Communication Polytechnic, Changsha 410132, China;
3. School of Construction Engineering, Xinyu University, Xinyu 338004, China

Abstract: As wavelet basis in wavelet analysis is neither arbitrary nor unique, the same signal dealing with different wavelet bases will generate different results. Therefore, how to construct a wavelet basis suitable for the characteristics of the analyzed signal and solve its algorithm and realization is a fundamental problem which perplexed many researchers. To solve these problems, in accordance with the basic features of the measured millisecond blast vibration signal, a new wavelet basis construction method based on the separation blast vibration signal is proposed, and the feasibility of this method is verified by comparing the practical effect of the newly constructed wavelet with other known wavelets in signal processing.

Key words: wavelet basis construction; curve fitting; millisecond blast vibration signal; sub-signal

1 Introduction

The primary purpose of wavelet theory study is to improve the efficiency and quality of signal processing. How to select or construct an appropriate wavelet for a given signal is a difficult task in wavelet analyzing [1−2]. Since using different wavelet functions to deal with the same signal will generate different results, the key to achieve a better effect in signal processing is choosing the optimal wavelet basis [3−4]. Although there are many known wavelet bases available at present, these wavelet bases cannot completely satisfy the requirements of signal processing. The practical effects of these wavelet bases in some signal analysis are still to be improved. Thus, it often needs to construct a new wavelet basis according to the measured signal in practical engineering [5].

Presently, existing wavelet construction methods are theoretically designing filters, lifting the existing algorithms, transforming wavelet bases, integrating the characteristics of several known wavelet functions and so on. Designing a filter to construct a wavelet is the earliest and most common method. Classic wavelets, such as Daubechies wavelets [6], Biorthgonal wavelets [7] and Polynomial Spline wavelets [8], are constructed by this method. VETTERLI and HERLEY [9] explored the relations among wavelets, filter banks, and multi-resolution signal analysis. On the basis of this work, STEFFEN et al [10], PHOONG et al [11], BAGGETT et al [12], BHARATH and NG [13] successfully constructed the regular M-band wavelet bases, a new class of two-channel biorthogonal filter banks and wavelet bases, non-MRA (Multi-resolution Analysis) wavelet bases and complex steerable wavelet bases, respectively. However, the work of designing a filter is complex and professional. It seems that only the mathematics or computer professionals are able to construct a wavelet basis. To simplify the wavelet construction process, the researchers made a lot of efforts. The lifting scheme proposed by SWELDENS [14] undoubtedly simplified the tedious process of wavelet construction, increased the flexibility of wavelet construction and contacted the wavelet theory more closely to practical application. Then, GOH [15], PIELLA and HEIJMANS [16], YANG et al [17] made further research on lifting algorithms to construct wavelet bases of different properties. With the developing of the wavelet design techniques, there are other more flexible ways to construct a wavelet basis in addition to the two classical wavelet construction methods mentioned above, such as transforming wavelet bases [18], integrating the characteristics of several known wavelet bases [19−20].

Though wavelet bases constructed by these known methods can strictly satisfy the admissibility conditions

Foundation item: Projects(51078043, 51278071, 51308072) supported by the National Natural Science Foundation of China
Received date: 2014−06−03; **Accepted date:** 2014−10−11
Corresponding author: ZHANG Sheng, Doctoral Candidate; Tel: +86−18942569648; E-mail: zhangsheng0403311@163.com

of the wavelet basis and the requirements of the optimal wavelet basis, the wavelet transform results of these wavelet bases can not highlight the characteristics of the analyzed signal very well due to the low similarity to the measured signal. Besides, these wavelet construction methods are mostly complex to get the derivation process and can only satisfy the requirements of the specified type of signal analysis. Therefore, it is imperative to seek a new wavelet construction method to construct a wavelet function that is highly similar to the characteristics of the measured signal. Moreover, this new wavelet construction method should have no complex derivation process and can be widely applied to various types of signal analysis.

In practice, if the main features of a signal are obtained, it will achieve a good effect by designing a matched wavelet basis to maximize the projection coefficients of the signal in some subspace of a wavelet function [21−22]. In this work, a new wavelet construction method is proposed and a new wavelet function that is highly similar to the measured signal is constructed by some simple mathematics methods. Furthermore, the feasibility and validity of this construction method are verified by applying the newly constructed wavelet basis to the measured millisecond blast vibration signal processing.

2 Wavelet basis construction method based on measured signal

2.1 A new wavelet construction method

Given $\psi(t)\in L^1(\mathbf{R})\cap L^2(\mathbf{R})$ and $\hat{\psi}(0)=0$, its Fourier transform is $\hat{\psi}(w)$. The function $\hat{\psi}(w)$ satisfies the admissible condition:

$$C_\psi=\int_{\mathbf{R}}\frac{|\hat{\psi}(w)|^2}{|w|}\mathrm{d}w<\infty \tag{1}$$

where $L^1(\mathbf{R})$ is absolutely integrable function space; $L^2(\mathbf{R})$ is square-integrable function space; w is the circular frequency. The function $\psi(t)$ is defined as basic wavelet or mother wavelet. After stretch and translation of the mother wavelet $\psi(t)$, a wavelet series can be given by

$$\psi_{a,b}(t)=\frac{1}{\sqrt{|a|}}\psi(\frac{t-b}{a}), a,b\in\mathbf{R};\ a\neq 0 \tag{2}$$

where a is the stretch factor (or the scale factor); b is the translation factor; the factor $|a|^{-1/2}$ is used to ensure energy conservation.

If the function $f(t)\in L^2(\mathbf{R})$ has finite energy, the continuous wavelet transform (CWT) of finite energy function $f(t)$ is defined as [23−24]

$$W_f(a,b)=\left\langle f(t),\psi_{a,b}(t)\right\rangle=\int_{-\infty}^{+\infty}f(t)\psi^*_{a,b}(t)\mathrm{d}t \tag{3}$$

where W_f (a,b) is wavelet transform coefficients; $\left\langle f(t),\psi_{a,b}(t)\right\rangle$ represents the inner product of the function $f(t)$ and the wavelet series $\psi_{a,b}(t)$; $\psi^*_{a,b}(t)$ denotes the complex conjugation of the wavelet series $\psi_{a,b}(t)$.

It can be seen from Eq. (3) that the projection coefficients of the function $f(t)$ on the wavelet series $\psi_{a,b}(t)$ is obtained by making a convolution operation between the function $f(t)$ and the wavelet series $\psi_{a,b}(t)$. The wavelet transform coefficients W_f (a,b) denote the similarity of the signal and the wavelet basis function at different scales and in different locations [25]. The features of the signal that are highly similar to the wavelet function waveform will be amplified, namely, the wavelet transform coefficients are relatively large. In contrast, the other features of the signal will be diminished, namely, the wavelet transform coefficients are relatively small. In other words, in order to make the characteristics of the signal reflected more clearly in the time-scale plane of wavelet, the projection coefficients of the signal should be as large as possible. To put it differently, the waveform of the wavelet function should be highly similar to the waveform of the analyzed signal, which is also the basic principle for the wavelet basis selection and construction.

In wavelet transform, the high similarity between the wavelet basis function and the analyzed signal can make its energy more concentrated after the wavelet transform, and also reduce the amount of calculation. According to the feature of the similarity between the wavelet basis function and the analyzed signal playing a very important role in improving the accuracy and resolution of the wavelet analysis results, a hypothesis of wavelet basis construction is proposed as follows. If the main characteristics of a measured signal can be extracted, a function that is highly similar to the characteristics of the measured signal can be constructed to satisfy the admissibility conditions of wavelet basis, which means that the function can be used as a new wavelet basis.

2.2 Feasibility analysis

As practical engineering signals are mostly complex and random, it is difficult to obtain a function that is highly similar to the measured signal. However, due to its particular generating conditions, millisecond blast vibration signal is unique among various geotechnical engineering signals. A millisecond blast vibration signal monitored at the measuring point is the superposition of several vibration waves induced by the explosion of detonators with different microsecond delay time. In millisecond blasting, under the assumption that the amplitude and frequency of vibration waves generated by the explosion of different detonators are roughly the same and the physical and mechanical properties of ore within the blasting range are similar, namely that the amplitude and frequency of vibration waves are approximately constant during the propagation process, each segment of vibration waves in millisecond blast vibration signal can be roughly expressed as the same time series with different microsecond delay time. In other words, a millisecond blast vibration signal monitored at the measuring point can be regarded as the superposition of several vibration waves. Each vibration wave is called sub-signal with the same amplitude, frequency and different microsecond delay time. Consequently, the waveform of sub-signal approximately reflects the overall trend of development of millisecond blast vibration signal, which means that the main features of millisecond blast vibration signal can be obtained by separating a sub-signal from the measured millisecond blast vibration signal.

An arbitrary function $\psi(t)\in L^2(\mathbf{R})$, which satisfies the volatility requirement $\int_R \psi(t)\mathrm{d}t = 0$ and attenuation requirement $\int_{-\infty}^{+\infty}(1+|t|^{\alpha})|\psi(t)|\mathrm{d}t < \infty$ that is applicable to a certain parameter α, is known as the mother wavelet or wavelet basis. A velocity−time curve of vibration sub-signal separated from the measured millisecond blast vibration velocity signal is illustrated in Fig. 1 [26]. Figure 1 shows that the amplitude of the velocity of sub-signal is alternating positive and negative variations and goes to zero in the shorter term, which basically satisfies the volatility and attenuation requirements of a wavelet basis function and preliminarily confirms that the function with high similarity to the sub-signal basically satisfies the admissible conditions of wavelet basis.

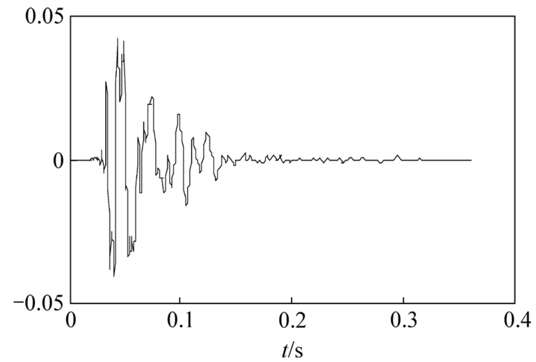

Fig. 1 Velocity−time curve of vibration sub-signal

Though any function belonging to $L^2(\mathbf{R})$ can be used as a wavelet basis function as long as it satisfies the admissible conditions of a wavelet basis, the function compactly supported or approximately compactly supported (time-domain locality) and with a certain regularity (frequency-domain locality) would be a better choice to enhance the localization analysis effect in time−frequency domain. The power spectra density of the sub-signal is shown in Fig. 2. As can be seen from Fig. 1 and Fig. 2, the sub-signal is both energy-concentrated and quick-decayed in time-domain and frequency-domain, which means that the sub-signal has a certain locality in time−frequency domain. Apparently, a wavelet basis function highly similar to the sub-signal has a good capability of time−frequency localization analysis.

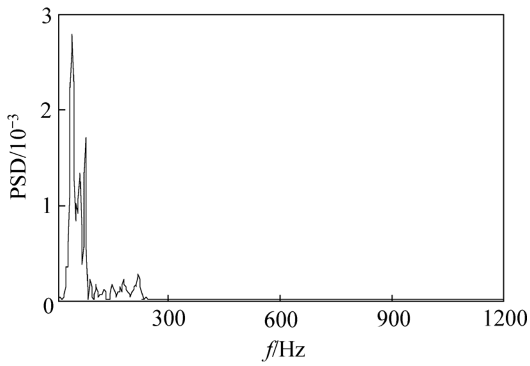

Fig. 2 Power spectra density of vibration sub-signal

3 Wavelet basis construction

By fitting the curve of sub-signal shown in Fig. 1 and adjusting the parameters of the fitting function, a new wavelet basis function is constructed. The specific steps are illustrated as follows:

1) Preprocess the vibration velocity sub-signal to improve the fitting accuracy before curve fitting. As shown in Fig. 1, when t>0.0776 s, the vibration velocity of sub-signal attenuates quickly and the amplitude becomes small, which can be excluded. Therefore, the main vibration interval (i.e. $0\le t\le 0.0776$ s) of the vibration velocity of sub-signal is selected for curve fitting.

2) Fit the curve of the preprocessed vibration velocity of sub-signal using curve fitting toolbox in MATLAB. By comparing the fitting coefficients of each function model, a Gaussian function model with the highest fitting coefficient is selected to fit the preprocessed vibration velocity of sub-signal by nonlinear least squares method. For convenience application of the fitting function in signal processing, it is necessary to adjust the parameters of the fitting function, and the final fitting function is expressed as follows:

$$\begin{aligned}f(x) = {} & 0.5193\exp[-((x-0.4348)/0.0125)^2]+\\ & 0.2566\exp[-((x-0.4968)/0.0090)^2]+\\ & 2.5330\exp[-((x-0.4725)/0.0593)^2]+\\ & 0.3743\exp[-((x-0.3408)/0.0109)^2]-\\ & 2.2270\exp[-((x-0.4712)/0.0762)^2]-\\ & 0.2316\exp[-((x-0.5897)/(-0.0151))^2]+\\ & 0.3026\exp[-((x-0.6356)/0.0255)^2]\end{aligned} \tag{4}$$

3) Verify whether the final fitting function $f(x)$ satisfies the volatility and attenuation requirements of a

wavelet basis function. With the help of MATLAB, it is obtained that $\int_{-\infty}^{+\infty} f(x)\mathrm{d}x = 0.0042$ and $\int_{-\infty}^{+\infty}(1+|x|)|f(x)|\mathrm{d}x = 0.8544$. For the reason that the integral value is relatively small to the function amplitude, it can be determined that the final fitting function $f(t)$ almost satisfies the volatility requirement (i.e. $\int_{-\infty}^{+\infty} f(x)\mathrm{d}x \approx 0$) and the attenuation requirement (i.e. $\int_{-\infty}^{+\infty}(1+|x|)|f(x)|\mathrm{d}x < \infty$) of the wavelet basis function. Thus, the function $f(t)$ can be used as a new wavelet basis function (as shown in Fig. 3).

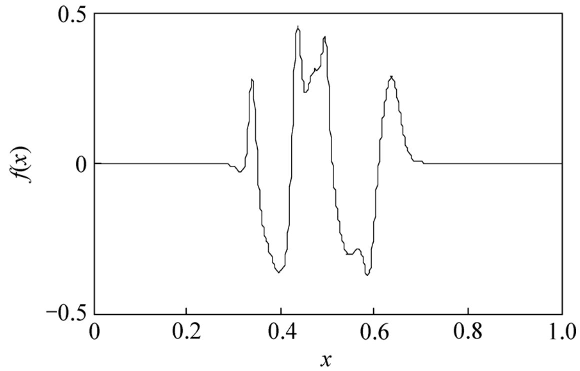

Fig. 3 Newly constructed wavelet basis function

4) Add the function $f(t)$ to the wavelet toolbox in MATLAB as a new wavelet basis function. An effective supported interval of the newly constructed wavelet basis function is [0 1], which is similar to the known Morlet wavelet and Mexican wavelet. The newly constructed wavelet basis function has neither a FIR filter nor scaling function.

Based on the several steps mentioned above, a flowchart of constructing a new wavelet basis function by this wavelet construction method is illustrated in Fig. 4. Different wavelet bases highly similar to the characteristics of any other type of the analyzed signal can be constructed by this flowchart in the future.

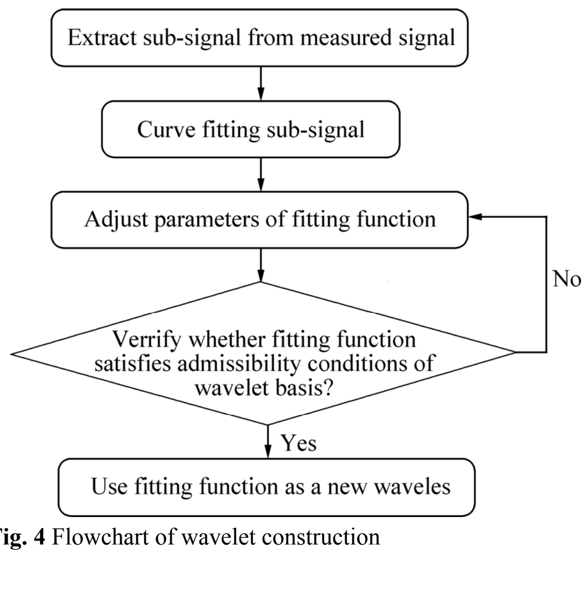

Fig. 4 Flowchart of wavelet construction

4 Applications

4.1 Accuracy test

The work done above has been successfully constructed a new wavelet basis function that is highly similar to the measured blast vibration signal. The merits of a wavelet basis function are mainly judged by its practical effects in signal analysis. However, before the newly constructed wavelet is applied to the measured signal analysis, its accuracy should be verified. Until recently, some main approaches to verify the accuracy of a wavelet basis function are analyzing signal reconstruction errors, comparing the errors between wavelet transform analysis results and theoretical values, contrasting to the simulation results, and so on. Based on that, an experiment to verify the accuracy of the newly constructed wavelet is designed by comparing the actual delay time which is identified by the newly constructed wavelet to the preset delay time in the analyzed signal.

4.1.1 Experiment

A sub-signal of the measured millisecond blast vibration signal is selected as a reference signal. Four identical sub-signals are superimposed sequentially at the same interval of 0.01 s from the start time of the reference signal to obtain the b10 signal, which means that the b10 signal is composed by five identical sub-signals and the delay time intervals of each adjacent sub-signal are 0.01 s. Velocity−time history curves of the reference sub-signal and the b10 signal are illustrated in Fig. 5.

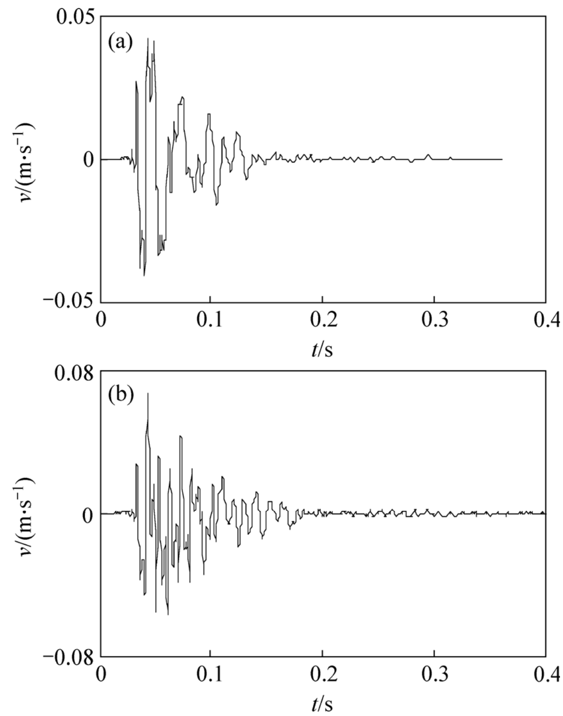

Fig. 5 Velocity−time curves of reference sub-signal (a) and b10 signal (b)

4.1.2 Results and analysis

In the wavelet analysis, wavelet transform modulus maximum method (WTMM) is often used to detect the mutational points of signals [27], which is for the reason that the mutational points of signals correspond to the modulus maximum of wavelet transform. Since the b10 signal is the superposition of four sub-signals based on the reference sub-signal, each superposition inevitably causes the mutation of the b10 signal. In other words, the mutation moments of the b10 signal are corresponding to the superposition time-point of each sub-signal. So, WTMM can be used to identify the moment of each superposition, and thus to determine the actual delay time intervals of each adjacent sub-signal in the b10 signal.

Conducting WTMM to the reference sub-signal and the b10 signal by using the newly constructed wavelet, the results are illustrated in Fig. 6.

1) Figure 6(a) shows that the local maximum of the wavelet transform modulus of the reference sub-signal appears at 0.042 s when the scale is 40.

2) Figure 6(b) shows that five local maxima of the wavelet transform modulus of the b10 signal appear at 0.042, 0.052, 0.062, 0.072, and 0.082 s respectively when the scale is 61. Comparing Fig. 6(b) and Fig. 6(a), it can be found that the first local maximum of the wavelet transform modulus of the b10 signal at 0.042 s is the modulus maximum of the reference sub-signal, and the other four are the modulus maximum of the sub-signal superposition. Setting the time when the modulus maximum of the reference sub-signal appears at 0.042 s as the initial time of the remaining four sub-signals superposition in the b10 signal, the superimposing moments of the four sub-signals superposition are calculated as 0.01, 0.02, 0.03, and 0.04 s respectively, namely, the delay time intervals of each adjacent sub-signal in the b10 signal are 0.01 s, which is identical to the preset delay time. The results prove that the newly constructed wavelet basis is effective in identifying the singular point of the analyzed signal.

4.2 Project analysis

Figure 7 shows the vibration velocity−time curve of a measured millisecond blast vibration signal. In order to determine the delay time intervals of adjacent detonators, WTMM method is conducted to the signal in Fig. 7. The wavelet transform modulus maxima are computed by using the newly constructed wavelet and db6 wavelet respectively, and the results are shown in Fig. 8.

As illustrated in Fig. 8(a), six local maxima of the wavelet transform modulus appear at 0.0680, 0.1076, 0.1980, 0.2724, 0.3188, and 0.3728 s respectively when the scale is 145. Since blasting of each detonator is a process of energy input and will inevitably arouse the mutation to the vibration velocity signal, the six time-points are determined by the newly constructed wavelet corresponding to the blasting initiation time of six detonators in this blasting. Defining the first local maximum of the wavelet transform modulus (i.e. 0.068 s) as the blasting initiation time of the minimum segment of detonators in this blasting, the microsecond delay time intervals of each segment of detonators are determined as 39.6, 90.4, 74.4, 46.4, and 54.0 ms, respectively.

As shown in Fig. 8(b), six local maxima of the

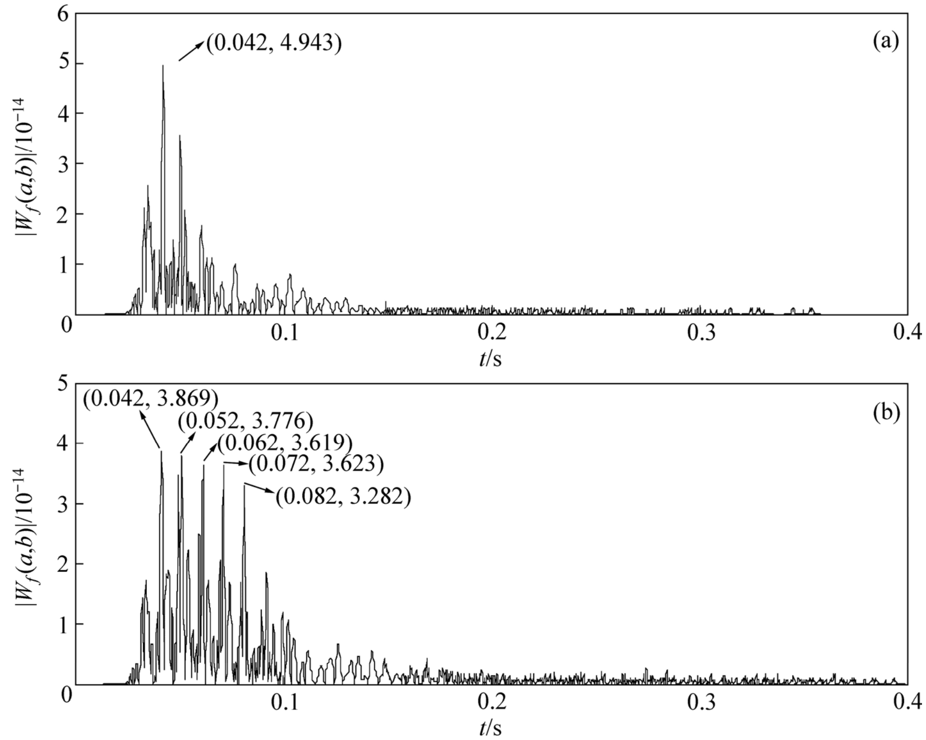

Fig. 6 Continuous wavelet transform modulus values of experimental signals: (a) Reference sub-signal (a=40); (b) b10 signal (a=61)

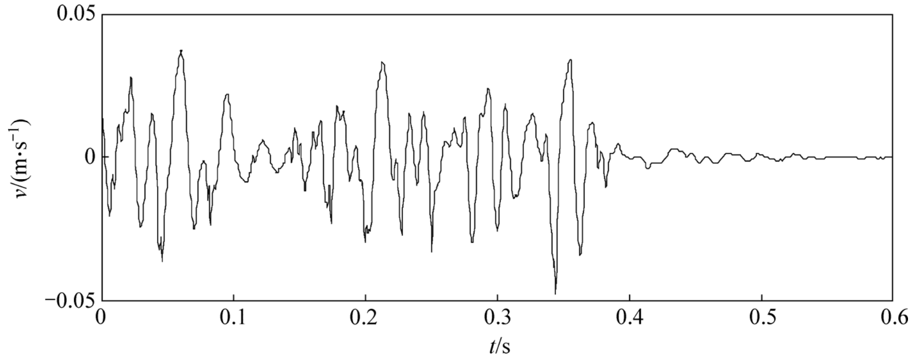

Fig. 7 Velocity−time curve of measured millisecond blast vibration signal

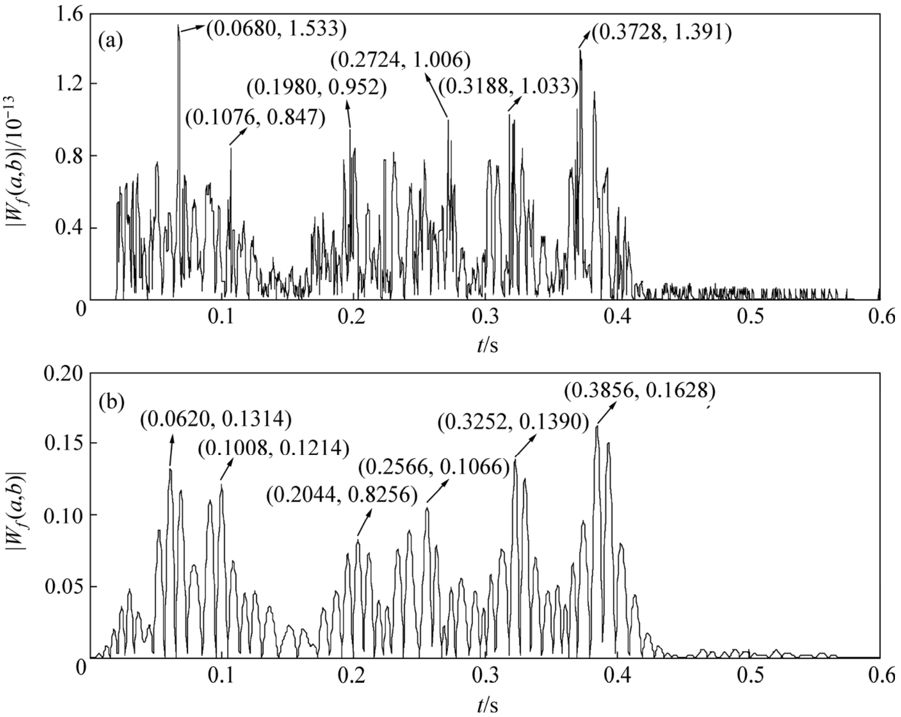

Fig. 8 Continuous wavelet transform modulus values of blast vibration signal: (a) Continuous wavelet transform modulus values determined by newly constructed wavelet (a=145); (b) Continuous wavelet transform modulus values determined by using db6 wavelet (a=28)

wavelet transform modulus appear at 0.062, 0.1008, 0.2044, 0.2568, 0.3252, and 0.3856 s respectively when the scale is 28, which are the blasting initiation time of six detonators determined by db6 wavelet in this blasting. Similarly, defining the first local maximum of the wavelet transform modulus (i.e. 0.062 s) as the blasting initiation time of the minimum segment of detonators, the microsecond delay time intervals of each segment of detonators are determined as 39.6, 90.4, 74.4, 46.4, and 54.0 ms, respectively.

Comparisons between the actual delay time intervals determined by two wavelet bases and the designed delay time intervals of detonators are illustrated in Table 1. It can be seen from Table 1 that the actual microsecond delay time intervals of 2−4, 5−6, 6−7 segment of detonators obtained by these two wavelets are within the designed interval limits; while the actual microsecond delay time intervals of 4−5, 7−9 segment of detonators determined by the two wavelets are beyond the designed interval limits, which indicates that it is necessary to be cautious to use the segments 4−5, 7−9 of this batch detonators in blasting. It is difficult to judge the pros and cons of the two wavelet bases by comparing the accuracy of determining delay time intervals.

Table 1 Comparison between designed delay intervals and actual delay intervals determined by two wavelets

Segment	Designed delay interval/ms	Actual delay interval/ms	
		Newly constructed wavelet	db6 wavelet
2−4	25−75	39.6	38.8
4−5	10−60	90.4	103.6
5−6	5−75	74.4	52.4
6−7	10−90	46.4	68.4
7−9	65−155	54.0	60.4

However, as shown in Fig. 8, it is clear that the resolution of wavelet transform analysis results obtained by the newly constructed wavelet basis is significantly higher than that obtained by db6 wavelet basis, which indicates that the newly constructed wavelet basis is more effective than db6 wavelet basis in detecting the singularity of blast vibration signal.

5 Conclusions

1) We have succeeded in proposing a new wavelet basis construction method based on the measured blast vibration signal. On this basis, a new wavelet function that is highly similar to the measured millisecond blast vibration signal has been constructed with the help of MATLAB. Moreover, the accuracy of the newly constructed wavelet basis in blast vibration analysis has been verified by using an experimental signal with the preset delay time. In addition, the effectiveness of the newly constructed wavelet basis in determining the delay time of millisecond blast vibration signal and detecting the singularity of the signal has been proved by comparing the application effect that is determined by the new wavelet and db6 wavelet.

2) The work done provides an effective way to improve the algorithm and implementation of existing wavelet basis construction methods. One of the great advantages of the new method is its simplicity. Engineering and technical personnel whose mathematical foundations are not very good can also construct wavelets that are applicable to the actual situations. Wavelets that are highly similar to various types of the analyzed signal can be constructed easier by this method in the future.

References

[1] CHAPA J O, RAO R M. Algorithms for designing wavelets to match a specified signal [J]. IEEE Transactions on Signal Processing, 2000, 48(12): 3395−3406.

[2] LU Cai-ping, DOU Lin-ming, LIU Hui, LIU Hai-shun, LIU Biao, DU Bin-bin. Case study on microseismic effect of coal and gas outburst process [J]. International Journal of Rock Mechanics & Mining Sciences, 2012, 53(9): 101−110.

[3] LU Wen-bo, LI Peng, CHEN Ming, ZHOU Chuang-bing, SHU Da-qiang. Comparison of vibrations induced by excavation of deep-buried cavern and open pit with method of bench blasting [J]. Journal of Central South University of Technology, 2011, 18(5): 1709−1718.

[4] ZHONG Guo-sheng, AO Li-ping, ZHAO Kui. Influence of explosion parameters on wavelet packet frequency band energy distribution of blast vibration [J]. Journal of Central South University, 2012, 19(9): 2674−2680.

[5] MAHESWARAN R, KHOSA R. Comparative study of different wavelets for hydrologic forecasting [J]. Computers & Geosciences, 2012, 46(3): 284−295.

[6] DAUBECHIES I. Orthonormal bases of compactly supported wavelets [J]. Communications on Pure and Applied Mathematics, 1988, 41(7): 909−996.

[7] COHEN A, DAUBECHIES I, FEAUVEAU J C. Biorthogonal bases of compactly supported wavelets [J]. Communications on Pure and Applied Mathematics, 1992, 45(5): 485−560.

[8] CHUI C K, WANG Jian-zhong. A cardinal spline approach to wavelets [J]. Proceedings of the American Mathematical Society, 1991, 113(3): 785−793.

[9] VETTERLI M, HERLEY C. Wavelets and filter banks: Theory and design [J]. IEEE Transactions on Signal Processing, 1992, 40(9): 2207−2232.

[10] STEFFEN P, HELLER P N, GOPINATH R A, BURRUS C S. Theory of regular M-band wavelet bases [J]. IEEE Transactions on Signal Processing, 1993, 41(12): 3497−3511.

[11] PHOONG S M, KIM C W, VAIDYANATHAN P P, ANSARI R. A new class of two-channel biorthogonal filter banks and wavelet bases [J]. IEEE Transactions on Signal Processing, 1995, 43(3): 649−665.

[12] BAGGETT L W, COURTER J E, MERRILL K D. The construction of wavelets from generalized conjugate mirror filters in $L^2(R^n)$ [J]. Applied and Computational Harmonic Analysis, 2002, 13(3): 201−223.

[13] BHARATH A A, NG J. A steerable complex wavelet construction and its application to image denoising [J]. IEEE Transactions on Image Processing, 2005, 14(7): 948−959.

[14] SWELDENS W. The lifting scheme: A custom-design construction of biorthogonal wavelets [J]. Applied and Computational Harmonic Analysis, 1996, 3(2): 186−200.

[15] GOH S S, JIANG Qing-tang, XIA Tao. Construction of biorthogonal multiwavelets using the lifting scheme [J]. Applied and Computational Harmonic Analysis, 2000, 9(3): 336−352.

[16] PIELLA G, HEIJMANS H J A M. Adaptive lifting schemes with perfect reconstruction [J]. IEEE Transcations on Signal Processing, 2002, 50(7): 1620−1630.

[17] YANG Xiao-yuan, SHI Yan, CHEN Liu-he, QUAN Zong-feng. The lifting scheme for wavelet bi-frames: Theory, structure, and algorithm [J]. IEEE Transcations on Image Processing, 2010, 19(3): 612−624.

[18] SAUCIER A. Construction of data-adaptive orthogonal wavelet bases with an extension of principal component analysis [J]. Applied and Computational Harmonic Analysis, 2005, 18(3): 300−328.

[19] ZHANG Jian-kang, DAVIDSON T N, WONG K M. Efficient design of orthonormal wavelet bases for signal representation [J]. IEEE Transactions on Signal Processing, 2004, 52(7): 1983−1996.

[20] MORILLAS P M. Construction of orthonormal wavelet-like bases [J]. Journal of Mathematical Physics, 2010, 51(8): 083510.

[21] BAHRAMPOUR A R, IZADNIA S, VAHEDI M. A variational method for designing wavelets to match a specified signal [J]. Signal Processing, 2008, 88(10): 2417−2424.

[22] BAHRAMPOUR A R, MIRZAEE S M A. A variational method for designing adaptive bandlimited wavelets [J]. Signal, Image and Video Processing, 2009, 3(4): 363−374.

[23] GALIANA-MERINO J J, ROSA-HERRANZ J L, ROSA-CINTAS S, MARTINEZ-ESPLA J J. Seismicwavetool: Continuous and discrete wavelet analysis and filtering for multichannel seismic data [J]. Computer Physics Communications, 2013, 184(1): 162−171.

[24] LIN Jing, QU Liang-sheng. Feature extraction based on morlet wavelet and its application for mechanical fault diagnosis [J]. Journal of Sound and Vibration, 2000, 234(1): 135−148.

[25] FRANGAKIS A S, STOSCHEK A, HEGERL R. Wavelet transform filtering and nonlinear anisotropic diffusion assessed for signal reconstruction performance on multidimensional biomedical data [J]. IEEE Transactions on Biomedical Engineering, 2001, 48(2): 213−222.

[26] LING Tong-hua, LI Xi-bing, WANG Gui-yao. A study on initiative control of blast vibration damages [J]. Rock and Soil Mechanics, 2007, 28(7): 1439−1442. (in Chinese)

[27] LING Tong-hua, ZHANG Sheng, CHEN Qian-qian, LIU Jia-shu. Pattern adapted wavelet construction and addition and its application in blast vibration signal analysis [J]. Journal of Vibration and Shock, 2014, 33(12): 53−57, 120. (in Chinese)

(Edited by YANG Hua)