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Optimization of sensing time and cooperative user allocation for OR-rule cooperative spectrum sensing in cognitive radio network

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Abstract: In order to improve the throughput of cognitive radio (CR), optimization of sensing time and cooperative user allocation for OR-rule cooperative spectrum sensing was investigated in a CR network that includes multiple users and one fusion center. The frame structure of cooperative spectrum sensing was divided into multiple transmission time slots and one sensing time slot consisting of local energy detection and cooperative overhead. An optimization problem was formulated to maximize the throughput of CR network, subject to the constraints of both false alarm probability and detection probability. A joint optimization algorithm of sensing time and number of users was proposed to solve this optimization problem with low time complexity. An allocation algorithm of cooperative users was proposed to preferentially allocate the users to the channels with high utilization probability. The simulation results show that the significant improvement on the throughput can be achieved through the proposed joint optimization and allocation algorithms.

Key words: cognitive radio; energy detection; cooperative spectrum sensing; throughput; optimization

1 Introduction

Due to the high-speed development of wireless communication technology and the rapid growth of radio users, the issue of scarce spectrum resources is becoming a serious problem [1]. Cognitive radio (CR) is proposed as an intelligent software radio for improving the utilization of the finite spectrum resources. CR can operate in the temporarily unused spectrum allocated to the primary user (PU) [2−3]. CR has to monitor the activation of the PU through continuously performing spectrum sensing in order to avoid causing harmful interference to the PU [4].

Energy detection is frequently used by CR because of its simple implementation without needing any prior information from the PU, but the hidden terminal problem may degrade its sensing performance [5]. Cooperative spectrum sensing is proposed to improve the sensing performance with the hidden terminal, where multiple CR users are designed to cooperatively detect the presence of the PU [6]. A fusion center is adopted to combine the sensing information from each CR user by a fusion rule such as hard fusion (i.e. AND rule, OR rule and K-OUT-N rule) and soft fusion [7]. In this work, we consider the OR rule that achieves the highest detection performance.

Sensing time becomes an important component in the design of cooperative spectrum sensing, as shorter sensing time degrades the detection performance while longer sensing time decreases the transmission time. Hence, for improving the throughput of the CR network, a sensing-throughput tradeoff model is proposed to obtain the optimal sensing time in Refs. [8−10]. However, in cooperative spectrum sensing, some of the time has to be spent on sending the sensing information from each CR user to the fusion center, which is called the cooperative overhead and related with the number of the cooperative users, and may decrease the transmission time greatly. Hence, the optimization of the cooperative users should also be considered. In Ref. [11], an optimal multi- channel cooperative spectrum sensing is proposed by formulating an optimization problem that maximizes the throughput of the CR while keeping the detection probability above a pre-defined threshold; however, the constraint of keeping the false alarm probability below a threshold is not considered. In Ref. [12], an optimization algorithm of the periodic cooperative spectrum sensing with the soft fusion is proposed; however, the soft fusion needs the CR users to transmit more sensing information, yielding the increase of the cooperative overhead, compared with the hard fusion. In addition, it is supposed [11−12] that the same number of users is allocated to each channel and do not give an optimal allocation

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algorithm of the CR users.

In this work, both the sensing time and the cooperative overhead are considered in the cooperative spectrum sensing with hard fusion. An optimization problem is formulated to obtain the optimal sensing time and cooperative user allocation through maximizing the throughput of the CR network is subject to of the probabilities of false alarm and detection. A joint optimization algorithm with lower time complexity is proposed to solve the proposed optimization problem. Then, an allocation algorithm is proposed to allocate the CR users to each channel based on the utilization probability and the received SNR.

2 Spectrum sensing technology

2.1 Energy detection

Energy detection is widely used in CR because of its reliable detection performance and low implementation cost. In energy detection, none of the prior knowledge of the PU signal is necessary, and the only need is to compare the energy of the received signal to a pre-defined threshold for obtaining a decision on the presence of the PU [10]. The spectrum sensing is processed as a binary hypothesis problem as follows:

$$y(t)=\begin{cases} n(t),\ H_0 \\ s(t)+n(t),\ H_1 \end{cases},\ t=1,2,\cdots,M \tag{1}$$

where the hypotheses H_0 and H_1 denote the absence and presence of the PU, respectively; $y(t)$ is the received sampling signal; $s(t)$ is the PU signal with the meaning of zero and the variance of σ_s^2; $n(t)$ is the Gaussian noise with the meaning of zero and the variance of σ_n^2; M denotes the number of samples and is given by

$$M=2\varepsilon f_s \tag{2}$$

where ε is the observation time and f_s is the sampling frequency. The energy statistic is given as

$$T(y)=\frac{1}{M}\sum_{t=1}^{M}|y(t)|^2 \tag{3}$$

If M is large enough, $T(y)$ approximately obeys the Gaussian distribution according to the center limit theorem (CLT) [10], and the probabilities of false alarm and detection are given as follows:

$$\begin{cases} P_f=Q\left(\left(\dfrac{\lambda}{\sigma_n^2}-1\right)\sqrt{\varepsilon f_s}\right) \\ P_d=Q\left(\left(\dfrac{\lambda}{\sigma_n^2}-\gamma-1\right)\sqrt{\dfrac{\varepsilon f_s}{2\gamma+1}}\right) \end{cases} \tag{4}$$

where λ is the threshold, and the function $Q(x)$ is defined as

$$Q(x)=\frac{1}{\sqrt{2\pi}}\int_x^{+\infty}\exp\left(-\frac{y^2}{2}\right)\mathrm{d}y \tag{5}$$

By supposing that $\bar{P}_f$ and $\bar{P}_d$ are respectively the upper limit of P_f and the lower limit of P_d, with $P_f \le \bar{P}_f$ and $P_d \ge \bar{P}_d$, λ is chosen as

$$\left(\frac{Q^{-1}(\bar{P}_f)}{\sqrt{\varepsilon f_s}}+1\right)\sigma_n^2 \le \lambda \le \left(Q^{-1}(\bar{P}_d)\sqrt{\frac{2\gamma+1}{\varepsilon f_s}}+\gamma+1\right)\sigma_n^2 \tag{6}$$

From Eq. (4), the false alarm probability is related to the detection probability as

$$P_f=Q(Q^{-1}(P_d)\sqrt{2\gamma+1}+\gamma\sqrt{\varepsilon f_s}) \tag{7}$$

2.2 Cooperative spectrum sensing

As shown in Fig. 1, we consider a CR network consisting of N CR users and a fusion center, and a PU network consisting of one PU base station and L available channels occupied by L PU terminals. Since there is a tall building on the transmission path between CR1 and the PU base station, the PU can be seen as a hidden terminal to CR1, and therefore the detection performance of CR1 is low because of receiving the weak power from the PU [13]. In order to improve the detection performance, multiple CR users are designed to cooperatively sense the PU, which is called cooperative spectrum sensing and performed by the following three steps [14].

1) Each CR user detects the PU by energy detection and makes a local binary decision as 0 (the absence of the PU) or 1 (the presence of the PU);

2) All the CR users send their local decisions to the fusion center through a public channel in their own time slots;

3) The fusion center combines the collected local decisions by AND rule, OR rule or K-OUT-N rule, and then obtains a final decision on the presence of the PU.

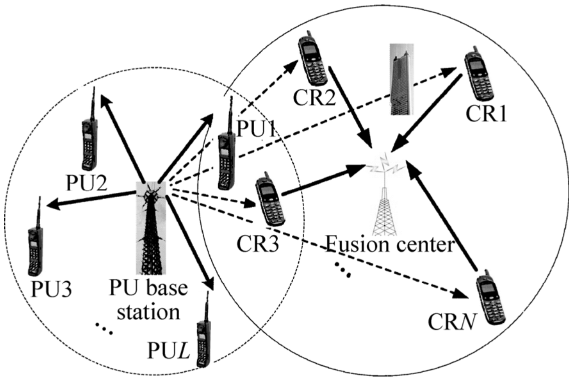

Fig. 1 CR and PU networks

In OR-rule cooperative spectrum sensing, the presence of the PU is finally decided if at least one CR user has detected the presence of the PU. However, in

AND-rule or K-OUT-N-rule cooperative spectrum sensing, the presence of the PU is determined only if multiple CR users detect the presence of the PU simultaneously. Hence, the detection probability of the OR-rule cooperative spectrum sensing is the largest, which decreases the interference to the PU greatly [15]. Hence, in this work only the OR-rule cooperative spectrum sensing is considered, whose probabilities of false alarm and detection are given as follows:

$$\begin{cases} Q_{\mathrm{f}} = 1-(1-P_{\mathrm{f},i})^N \\ Q_{\mathrm{d}} = 1-(1-P_{\mathrm{d},i})^N \end{cases} \tag{8}$$

where $P_{\mathrm{f},i}$ and $P_{\mathrm{d},i}$ are the probabilities of false alarm and detection of CR i, respectively.

3 Optimization scheme

3.1 Frame structure

The frame structure of OR-rule cooperative spectrum sensing is shown in Fig. 2, where each frame is divided into multiple transmission time slots and one sensing time slot. If an idle channel is detected, each CR user can use this channel to transmit data in its allocated time slot, and in the last time slot of the frame, cooperative spectrum sensing including local sensing and cooperative overhead is performed by all the CR users, in order to detect the appearance of the PU. In cooperative spectrum sensing, each CR user firstly obtains the local sensing result by energy detection and then sends the sensing result to a fusion center that combines all the received results to get a final decision by OR rule, in its mini time slot of cooperative overhead. If the final decision on the absence of the PU is made, the CR user can continue communication in the following frame; else, it has to vacate this channel and search for a new idle channel from the left channels in order to avoid causing interference to the PU.

3.2 Joint optimization algorithm

Supposing that the length of each frame is T, and the spectrum sensing time is T_{s}, the total data transmission time is given by

$$T_{\mathrm{d}} = T - T_{\mathrm{s}} \tag{9}$$

Assuming that the local sensing time is ε, the number of cooperative users is k, and the length of mini time slot is ξ, the spectrum sensing time is given by

$$T_{\mathrm{s}} = \varepsilon + k\xi \tag{10}$$

From Eqs. (9) and (10), the average throughput of the CR network in one frame is given by [16]

$$R(\varepsilon,k) = \frac{T-\varepsilon-k\xi}{T}(C_0 P_{H_0}(1-Q_{\mathrm{f}}) + C_1 P_{H_1}(1-Q_{\mathrm{d}})) \tag{11}$$

where P_{H_0} and P_{H_1} are the probabilities of H_0 and H_1, respectively; C_0 and C_1 are the rates of the CR user at the absence and presence of the PU, respectively .

Our goal is to formulate an optimization problem to maximize the average throughput of the CR network subject to the constraints of the probabilities of false alarm and detection as follows:

$$\begin{cases} \max\limits_{\varepsilon,k} R(\varepsilon,k) \\ \text{s.t.} \begin{cases} Q_{\mathrm{f}} \le \bar{Q}_{\mathrm{f}} \\ Q_{\mathrm{d}} \ge \bar{Q}_{\mathrm{d}} \\ \varepsilon + k\xi \le T \\ 1 \le k \le N, k \in Z \end{cases} \end{cases} \tag{12}$$

where $\bar{Q}_{\mathrm{f}} \le 0.5$ and $\bar{Q}_{\mathrm{d}} \ge 0.5$ are the upper limit of false alarm probability and the lower limit of detection probability, respectively. For simplifying Eq. (12), we give the following **Lemma 1**.

Lemma 1: R can achieve the maximum only at $Q_{\mathrm{d}} = \bar{Q}_{\mathrm{d}}$.

Proof: We prove that Q_{f} increases with the increase of Q_{d}. From (8), the one-order derivative of Q_{f} in Q_{d}

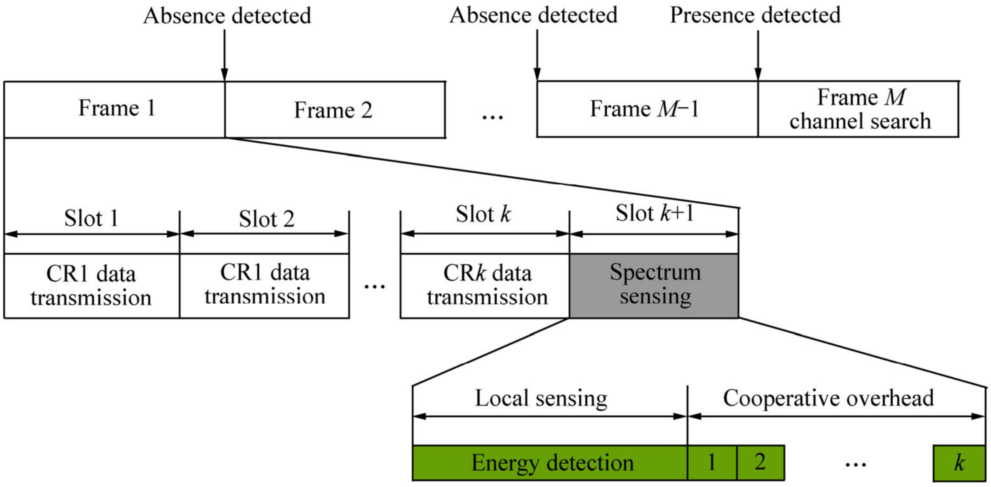

Fig. 2 Frame structure of OR-rule cooperative spectrum sensing

is given by

$$\frac{\partial Q_{\mathrm{f}}}{\partial Q_{\mathrm{d}}}=\frac{\partial Q_{\mathrm{f}}/\partial P_{\mathrm{f},i}}{\partial Q_{\mathrm{d}}/\partial P_{\mathrm{d},i}}\times\frac{\partial P_{\mathrm{f},i}/\partial\lambda}{\partial P_{\mathrm{d},i}/\partial\lambda}=\frac{1-P_{\mathrm{f},i}}{1-P_{\mathrm{d},i}}\times\frac{\partial P_{\mathrm{f},i}/\partial\lambda}{\partial P_{\mathrm{d},i}/\partial\lambda} \tag{13}$$

From Eq. (4), we have

$$\frac{\partial P_{\mathrm{f},i}/\partial\lambda}{\partial P_{\mathrm{d},i}/\partial\lambda}=\frac{1}{\sqrt{2\gamma_i+1}}\times \exp\left(\frac{M}{2}\left(\frac{\lambda}{\sigma_{\mathrm{n}}^2}-1\right)^2-\frac{M}{4\gamma_i+2}\left(\frac{\lambda}{\sigma_{\mathrm{n}}^2}-\gamma_i-1\right)^2\right) \tag{14}$$

which indicates that $(\partial P_{\mathrm{f},i}/\partial\lambda)/(\partial P_{\mathrm{d},i}/\partial\lambda)>0$. Hence, from Eq. (13) we have $\partial Q_{\mathrm{f}}/\partial Q_{\mathrm{d}}>0$, which proves that Q_{f} increases with the increase of Q_{d}. Assuming that $Q_{\mathrm{f}}=\tilde{Q}_{\mathrm{f}}$ at $Q_{\mathrm{d}}=\bar{Q}_{\mathrm{d}}$, we have $Q_{\mathrm{f}}>\tilde{Q}_{\mathrm{f}}$ at $Q_{\mathrm{d}}>\bar{Q}_{\mathrm{d}}$ which indicates $1-Q_{\mathrm{d}}<1-\bar{Q}_{\mathrm{d}}$ and $1-Q_{\mathrm{f}}<1-\tilde{Q}_{\mathrm{f}}$. Hence from Eq. (11), R decreases with the increase of Q_{d} and achieves the maximum only at $Q_{\mathrm{d}}=\bar{Q}_{\mathrm{d}}$. End proof.

With $Q_{\mathrm{d}}=\bar{Q}_{\mathrm{d}}$, the detection probability of each CR user is given by

$$P_{\mathrm{d},i}=1-(1-\bar{Q}_{\mathrm{d}})^{\frac{1}{k}} \tag{15}$$

By substituting Eq. (15) into Eq. (7), the false alarm probability is given by

$$Q_{\mathrm{f}}=1-\left(1-Q\left(\sqrt{2\gamma_i+1}Q^{-1}\left(1-(1-\bar{Q}_{\mathrm{d}})^{\frac{1}{k}}\right)+\gamma_i\sqrt{\varepsilon f_{\mathrm{s}}}\right)\right)^k \tag{16}$$

By substituting Eq. (16) into Eq. (11), the average throughput at $Q_{\mathrm{d}}=\bar{Q}_{\mathrm{d}}$ is rewritten as

$$R(\varepsilon,k)=\frac{T-\varepsilon-k\xi}{T}\left[\rho\left(1-Q\left(\alpha(k)+\gamma_i\sqrt{\varepsilon f_{\mathrm{s}}}\right)\right)^k+\eta\right] \tag{17}$$

where $\rho=C_0P_{H_0}$, $\eta=C_1P_{H_1}(1-\bar{Q}_{\mathrm{d}})$ and $\alpha(k)$ is given by

$$\alpha(k)=\sqrt{2\gamma_i+1}Q^{-1}\left(1-(1-\bar{Q}_{\mathrm{d}})^{\frac{1}{k}}\right) \tag{18}$$

According to the first constraint of Eq. (12), we need $Q_{\mathrm{f}}\le\bar{Q}_{\mathrm{f}}$ as

$$Q_{\mathrm{f}}=1-\left(1-Q\left(\alpha(k)+\gamma_i\sqrt{\varepsilon f_s}\right)\right)^k\le\bar{Q}_{\mathrm{f}} \tag{19}$$

where we obtain:

$$\varepsilon\ge\frac{(\beta(k)-\alpha(k))^2}{\gamma_i^2 f_{\mathrm{s}}} \tag{20}$$

where $\beta(k)=Q^{-1}\left(1-\left(1-\bar{Q}_{\mathrm{f}}\right)^{\frac{1}{k}}\right)$. From Eqs. (16) to (20), the optimization problem Eq. (12) is simply rewritten as follows:

$$\begin{cases}\max\limits_{\varepsilon,k} R(\varepsilon,k)=\dfrac{T-\varepsilon-k\xi}{T}\cdot \\ \qquad\left(\rho\left(1-Q\left(\alpha(k)+\gamma_i\sqrt{\varepsilon f_{\mathrm{s}}}\right)\right)^k+\eta\right) \\ \text{s.t.}\ \begin{cases}G(k)\le\varepsilon\le T-k\xi\\ 1\le k\le N, k\in Z\end{cases}\end{cases} \tag{21}$$

where $G(k)=(\beta(k)-\alpha(k))^2/(\gamma_i^2 f_{\mathrm{s}})$. For solving Eq. (21), we give the following **Lemma 2**.

Lemma 2: With the given k, there is an optimal $\varepsilon_0\in[0, T-k\xi]$ that maximizes $R(\varepsilon,k)$.

Proof: Take the one-order partial derivative of R in ε as follows:

$$\nabla R(\varepsilon,k)=-\frac{1}{T}\left(\rho\left(1-Q\left(\alpha(k)+\gamma_i\sqrt{\varepsilon f_{\mathrm{s}}}\right)\right)^k+\eta\right)+ k\rho\frac{T-k\xi-\varepsilon}{2T\sqrt{2\pi\varepsilon f_{\mathrm{s}}}}\left(1-Q\left(\alpha(k)+\gamma_i\sqrt{\varepsilon f_{\mathrm{s}}}\right)\right)^{k-1}\times \exp\left(-\frac{\left(\alpha(k)+\gamma_i\sqrt{\varepsilon f_{\mathrm{s}}}\right)^2}{2}\right) \tag{22}$$

Since $1-Q(x)\ge0$, we have

$$\begin{cases}\lim\limits_{\varepsilon\to0}\nabla R=O\left(\dfrac{T-k\xi}{2T\sqrt{2\pi\varepsilon f_{\mathrm{s}}}}\right)=+\infty\\ \lim\limits_{\varepsilon\to T-k\xi}\nabla R=-\dfrac{\left(\rho\left(1-Q\left(\alpha(k)+\gamma_i\sqrt{\varepsilon f_{\mathrm{s}}}\right)\right)^k+\eta\right)}{T}<0\end{cases} \tag{23}$$

where $O(x)$ is the dimension of x. Equation (23) indicates that R firstly increases with $\varepsilon\to0$ and then decreases with $\varepsilon\to T-k\xi$. Hence, R is a convex function and has the maximum, where $\varepsilon_0\in[0, T-k\xi]$ that makes $\nabla R(\varepsilon_0, k)=0$ is the maximal point. End proof.

Since the actual bound of ε is $[G(k), T-k\xi]$, we have to discuss the optimal solution ε^* of Eq. (21) based on the different conditions. If $G(k)\in[0, \varepsilon_0]$, the optimal solution $\varepsilon^*=\varepsilon_0$ is shown in Fig. 3(a); while if $G(k)\in[\varepsilon_0, T-k\xi]$, the optimal solution $\varepsilon^*=G(k)$ is shown in Fig. 3(b).

When the optimal ε^* is obtained, we need to get the optimal number of cooperative users k^*. Since k is an integer from 1 to N, we can obtain k^* by the enumeration method. However, sometimes N may be very large, and from the constraints of Eq. (21), we have

$$\begin{cases}T-k\xi\ge0\\ T-k\xi\ge\dfrac{(\beta(k)-\alpha(k))^2}{\gamma_i^2 f_{\mathrm{s}}}\\ 1\le k\le N\end{cases} \tag{24}$$

where the search bound of k is specified as follows:

$$\max\left(1,\left\lfloor\frac{\ln(1-Q_{\mathrm{d}})}{\ln\left(1-Q\left(-\gamma_i\sqrt{\dfrac{\varepsilon f_{\mathrm{s}}}{2\gamma_i+1}}\right)\right)}\right\rfloor\right)\le k\le \min\left(N,\left\lceil\frac{T-\varepsilon}{\xi}\right\rceil\right) \tag{25}$$

where $\lfloor x\rfloor$ and $\lceil x\rceil$ denote the minimal integer above x and the maximal integer below x, respectively.

For solving Eq. (21), the alternating direction optimization is adopted, where we firstly optimize ε with the given k and then optimize k with the given ε. This optimization process is alternately implemented until R is convergent [17]. According to the **Lemma 2**, we know that R is convergent when ε approaches to the optimal value. The joint optimization of sensing time and number of cooperative uses is shown in the **Algorithm 1**, where we array the CR users in descending order of their SNRs, because the user with large SNR can obtain high detection performance and should be preferentially chosen [18]. In addition, the symbol function sign(x) is used to distinguish the two conditions of Fig. 3.

In Algorithm 1, when k is given, we search ε beginning from the middle of the sequence, namely, if the derivative of R in the current search position is above zero, we search ε from the front half part; else, we search ε from the last half part. Hence, half of the data is discarded to save the search time. Supposing that the estimation accuracy is δ, the time complexity of searching ε with given k is $O(\log_2(T/\delta))$. Since we search k from $k_{\min}$ to $k_{\max}$, the total time complexity of the Algorithm 1 is $O((k_{\max}-k_{\min})\log_2(T/\delta))$, that is lower than $O(N\log_2(T/\delta))$. Hence, the time complexity of the exhaustive search denoted by $O(NT/\delta)$ is larger than that of the **Algorithm 1**.

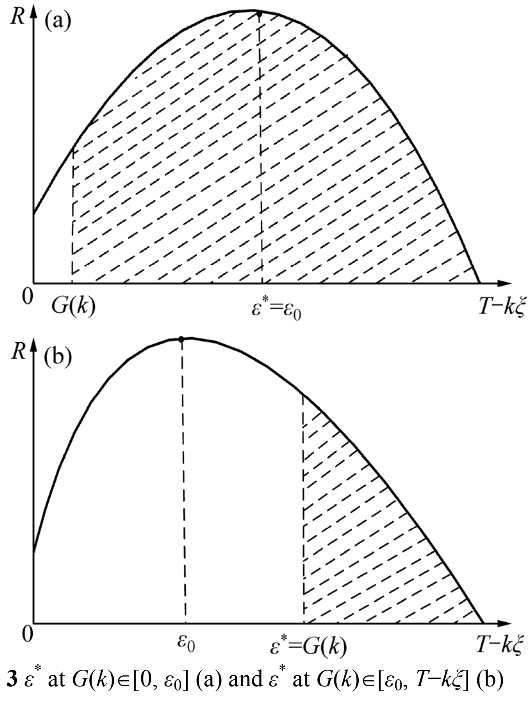

Fig. 3 ε^* at $G(k)\in[0, \varepsilon_0]$ (a) and ε^* at $G(k)\in[\varepsilon_0, T-k\xi]$ (b)

When the optimal ε^* and k^* are both obtained, the total data transmission time in a frame is $T_{\mathrm{d}}=T-\varepsilon^*-k^*\xi$. The transmission time of each CR user is allocated according to the QoS, which is given by

$$t_{\mathrm{d},n}=\frac{(T-\varepsilon^*-k^*\xi)\vartheta_n}{\sum\limits_{i=1}^{k}\vartheta_i},\ n=1,2,\cdots,k^* \tag{26}$$

where ϑ_i is the QoS of CR i.

Noting that the goal of optimizing Eq. (12) is to achieve the globally maximum of the objective function, the value of the objective function in each iteration of the **Algorithm 1** is non-decreasing as follows:

$$R(\varepsilon^{(i-1)},k^{(i-1)})\le R(\varepsilon^{(i-1)},k^{(i)})\le R(\varepsilon^{(i)},k^{(i)}) \tag{27}$$

which indicates that if $R(\varepsilon,k)$ is convergent, both of ε and k must be convergent as $\left|\varepsilon^{(i)}-\varepsilon^{(i-1)}\right|\le\delta$ and $\left|k^{(i)}-k^{(i-1)}\right|\le\delta$.

```
Algorithm 1: Joint optimization algorithm
1) Initialization: set the estimation accuracy $\delta=10^{-3}$ and the iterative number $i$=0; set $k^{(i)}=1$ and $\tau^{(i)}=0$;
2) With the given $k^{(i)}$, set $\varepsilon_{\min}=G(k^{(i)})$ and $\varepsilon_{\max}=T-k^{(i)}\xi$;
3) If sign($\nabla R(\varepsilon_{\min}, k^{(i)})$)≡sign($\nabla R(\varepsilon_{\max}, k^{(i)})$): let $\varepsilon^{(i+1)}=\varepsilon_{\min}$ and go to step 7); else if sign($\nabla R(\varepsilon_{\min}, k^{(i)})$)≡sign($\nabla R(0, k^{(i)})$): continue;
4) While $|\varepsilon_{\max}-\varepsilon_{\min}|>\delta$ do:
    (a) Let $\bar{\varepsilon}=(\varepsilon_{\min}+\varepsilon_{\max})/2$;
    (b) If $\mathrm{sign}(\nabla R(\varepsilon_{\min},k^{(i+1)}))\equiv\mathrm{sign}(\nabla R(\bar{\varepsilon},k^{(i+1)}))$: $\varepsilon_{\min}=\bar{\varepsilon}$ and continue;
    (c) If $\mathrm{sign}(\nabla R(\varepsilon_{\max},k^{(i+1)}))\equiv\mathrm{sign}(\nabla R(\bar{\varepsilon},k^{(i+1)}))$: $\varepsilon_{\max}=\bar{\varepsilon}$ and continue;
5) Let $\varepsilon_0=\frac{1}{2}(\varepsilon_{\max}+\varepsilon_{\min})$ and $\varepsilon^{(i+1)}=\varepsilon_0$;
6) With the given $\varepsilon^{(i+1)}$, calculate the lower and the upper bounds of $k$ as $k_{\min}$ and $k_{\max}$ according to Eq. (25);
7) Search $k^{(i+1)}=\mathrm{argmax}R(\varepsilon^{(i+1)}, k)$ for $k_{\min}\le k\le k_{\max}$ and set $i=i+1$;
8) Repeat Steps 2) to 7) until $|R(\varepsilon^{(i)},k^{(i)})-R(\varepsilon^{(i-1)},k^{(i-1)})|\le\delta$;
9) Output: the optimal solution $\varepsilon^*=\varepsilon^{(i)}$ and $k^*=k^{(i)}$.
```

4 Allocation algorithm of cooperative users

4.1 Channel utilization probability

If the CR users detect the presence of the PU, they have to vacate this channel, and search another new idle channel from the left $L-1$ channels and switch to this idle channel to continue communication. The idle probability of detecting one channel is given as follows:

$$P_{\mathrm{I}} = P_{H_0}(1-Q_{\mathrm{f}}) + P_{H_1}(1-Q_{\mathrm{d}}) \tag{28}$$

where the average time to search an idle channel from the left $L-1$ channels is given by [16]

$$T_{\mathrm{f}} = T_{\mathrm{s}}P_{\mathrm{I}} + 2T_{\mathrm{s}}(1-P_{\mathrm{I}})P_{\mathrm{I}} + \cdots + (L-1)T_{\mathrm{s}}(1-P_{\mathrm{I}})^{L-2}P_{\mathrm{I}} \tag{29}$$

After mathematical derivation, Eq. (29) is further rewritten as

$$T_{\mathrm{f}} = T_{\mathrm{s}}\left[\frac{1-(1-P_{\mathrm{I}})^{L}}{P_{\mathrm{I}}} - L(1-P_{\mathrm{I}})^{L-1}\right] \tag{30}$$

In order to guarantee to find an idle channel, the probability that all the $L-1$ channels are busy should be very small as follows:

$$(1-P_{\mathrm{I}})^{L-1} \le \varphi << 1 \tag{31}$$

To avoid causing interference to the PU, we need $Q_{\mathrm{d}} \approx 1$ and then have $P_{\mathrm{I}} \approx P_{H_0}(1-Q_{\mathrm{f}})$. From Eq. (31), we can obtain that

$$Q_{\mathrm{f}} \le 1 - \frac{1}{P_{H_0}}(1-\sqrt[L-1]{\varphi}) \tag{32}$$

With $Q_{\mathrm{f}} \le \bar{Q}_{\mathrm{f}}$, Eq. (32) can be tenable by supposing $\bar{Q}_{\mathrm{f}} \le 1-(1-\sqrt[L-1]{\varphi})/P_{H_0}$. By substituting Eq. (31) into Eq. (30), T_{f} is simply written as

$$T_{\mathrm{f}} \approx \frac{T_{\mathrm{s}}}{P_{\mathrm{I}}} \approx \frac{T_{\mathrm{s}}}{P_{H_0}(1-Q_{\mathrm{f}})} \tag{33}$$

After the data transmission of a frame, the CR users have to spend the search time T_{f} in the probability $1-P_{\mathrm{I}}$. The utilization probability of one channel is defined as

$$P_{\mathrm{U}} = \frac{T-T_{\mathrm{s}}}{T+(1-P_{\mathrm{I}})T_{\mathrm{f}}} = \frac{(T-\varepsilon-k\xi)(1-Q_{\mathrm{f}})}{T(1-Q_{\mathrm{f}})+(1/P_{H_0}-(1-Q_{\mathrm{f}}))(\varepsilon+k\xi)} \tag{34}$$

4.2 Allocation algorithm

Algorithm 1 confirms the sensing time and the number of cooperative users in each channel. The following step is to allocate all the N CR users to the L channels in order to obtain the maximal benefit. The allocation scheme must satisfy the following two requirements:

1) Each CR user should preferentially choose the channel with high utilization probability in order to improve its spectrum utilization;

2) Each channel should choose the CR users with high received SNRs in order to improve the cooperative detection performance.

Supposing that the received SNR of CR n in channel l is $\gamma_{n,l}$ for n=1, 2, ⋯, N and l=1, 2, ⋯, L, and the optimal number of cooperative users in channel l is k_l, the allocation of CR users is shown in **Algorithm 2,** where the CR users is preferentially allocated to the channel with larger utilization probability in step 2). If the fusion decision at the end of one frame is on the presence of the PU, the CR users in this busy channel must vacate to find another idle channel in the following frame. The number of the left channels is $L_{\mathrm{m}}=L-L_{\mathrm{c}}-1$ where L_{c} is the number of the channels occupied by the other CR users. If there is no idle channel detected from the left L_{m} channels, the CR users in the busy channel should be allocated to the L_{c} occupied idle channels in the step 4), namely, the CR user should be allocated to the idle channel where it has the largest received SNR. However, if there are some idle channels detected from the left L_{m} channels, the CR users are allocated to these idle channels in step 3).

Algorithm 2: Allocation of CR users

```
1) Initialization: define the set of $N$ CRs as $\boldsymbol{A}=\{\mathrm{CR}_n\}_{n=1}^{N}$, the set of $L$ channels as $\boldsymbol{B}=\{\mathrm{Ch}_l\}_{l=1}^{L}$ and the utilization probabilities of $L$ channels as $P_{\mathrm{U},l}$ for $l$=1,2,⋯,$L$;
2) Array the channels of $\boldsymbol{B}$ in descending order of their utilization probabilities $P_{\mathrm{U},l}$;
3) For $l$=1 to $L$ do:
    (a) Array the CR users of $\boldsymbol{A}$ in descending order of their received SNR in channel $l$ $\gamma_{n,l}$;
    (b) $\mathrm{Ch}_l$ chooses the front $k_l$ CR users of $\boldsymbol{A}$ as the set $\boldsymbol{C}_l=\{\mathrm{CR}_n\}_{n=1}^{k_l}$;
    (c) Let $\boldsymbol{A}=\boldsymbol{A}-\boldsymbol{C}_l$ and $N=N-k_l$;
    (d) If $\boldsymbol{A}=\varnothing$: break and go to step 5), else continue.
4) While $\boldsymbol{A}=\varnothing$ do:
    (a) With $\{\mathrm{CR}_n\}\in\boldsymbol{A}$ search $l^{*}=\arg\max\gamma_{n,l}$ for $l$=1, 2, ⋯, $L$;
    (b) Let $\boldsymbol{C}_{l^{*}}=\boldsymbol{C}_{l^{*}}\cup\{\mathrm{CR}_n\}$;
    (c) $\boldsymbol{A}=\boldsymbol{A}-\{\mathrm{CR}_n\}$.
5) Output: the user set of each channel $\boldsymbol{C}_l$ for $l$=1, 2, ⋯, $L$.
```

5 Simulation results

In the simulations, we suppose that the total number of CR users N=20, the number of channels L=4, the frame length T=5 ms, the sampling frequency f_{s}=1 GHz, the upper limit of false alarm probability and the lower limit of detection probability Q_{f}=0.1 and Q_{d}=0.99 respectively, the length of mini time slot ξ=0.1 ms, the average rates at the absence and presence of the PU C_0=25 kbps and C_1=8 kbps, respectively, and the probabilities of H_0 and H_1, $P_{H_0}=0.4$ and $P_{H_1}=0.6$, respectively.

Figure 4 indicates the throughput R with different sensing time ε and number of users k. It is obvious that there exists the optimal ε and k that maximizes R, and the convex surface of R validates the correctness of the **Lemma 2** and the convergence of **Algorithm 1**.

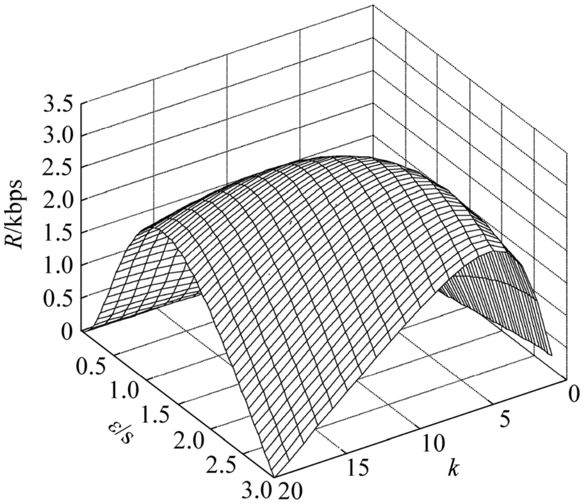

Fig. 4 Throughput with different sensing time and numbers of users

Figure 5 shows the throughput versus the number of users with different sensing time when SNR=−5dB. We obtain the optimal k=8 by **Algorithm 1**. It is seen that both the throughputs of k=8 and 20 are low at small or large ε, because small ε degrades the false alarm probability while large ε decreases the transmission time. It is also known that the maximal throughput of k=8 obtained by **Algorithm 1** is larger than those of k=1 and 20, because small k decreases the detection performance while large k increases the cooperative overhead.

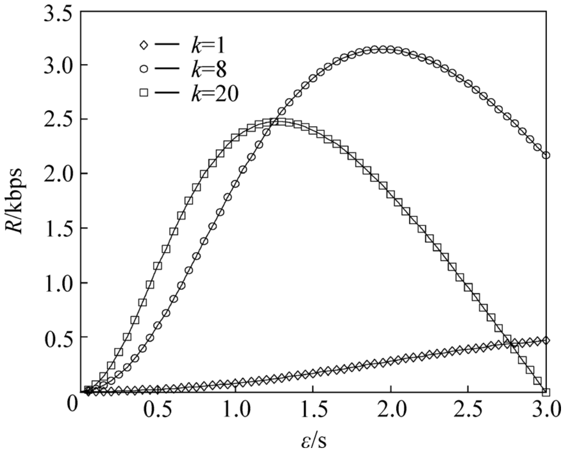

Fig. 5 Throughput versus number of users with different sensing time

Figure 6 indicates the throughput versus the three different algorithms: the proposed joint optimization algorithm, the sensing-throughput tradeoff algorithm [8], and the single-user detection. It is seen that the proposed algorithm outperforms the other two by producing lower cooperative overhead and longer transmission time, subject to the constraint of the detection performance. It is also seen that compared with the single-user detection, the throughput of the sensing-throughput tradeoff algorithm is large at low SNR and small at high SNR, because the performance of the single-user detection is high enough at large SNR; however, the cooperative overhead of the single-user detection is much lower than that of the sensing-throughput tradeoff algorithm.

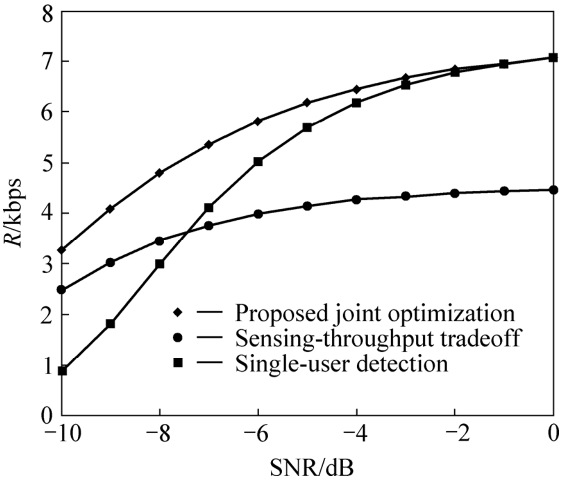

Fig. 6 Throughput versus different algorithms

Figure 7 shows the throughput R versus the utilization probability P_U=0.8, 0.6, 0.4 and 0.2, with different k values. The maximal throughput of each channel and the average throughput of each CR user with different P_U are listed in Table 1. From Table 1, it can be seen that the channel with P_U=0.8 can achieve the largest throughput with the least CR users, namely, the average throughput of each CR user in this channel is the largest. However, each CR user in the channel with P_U=0.2 obtains the smallest average throughput because the cooperative overhead increases with more users. Hence, we preferentially allocate the users to the channel with higher P_U in **Algorithm 2**.

Figure 8 indicates the throughput versus the different orders of the received SNR of the CR users. It is seen that the throughput with the descending order of SNR is larger than that with the ascending order of SNR because the CR users with high SNR are preferentially allocated to the channels through the descending order of SNR, yielding higher spectrum sensing performance. Figure 9 shows the throughput comparison between the proposed allocation algorithm and the average allocation algorithm that allocates the same number of users to each channel. It can be seen that the proposed allocation algorithm can achieve the larger throughput because in the average allocation algorithm, more users are allocated to the channel with high P_U, yielding the increase of the cooperative overhead.

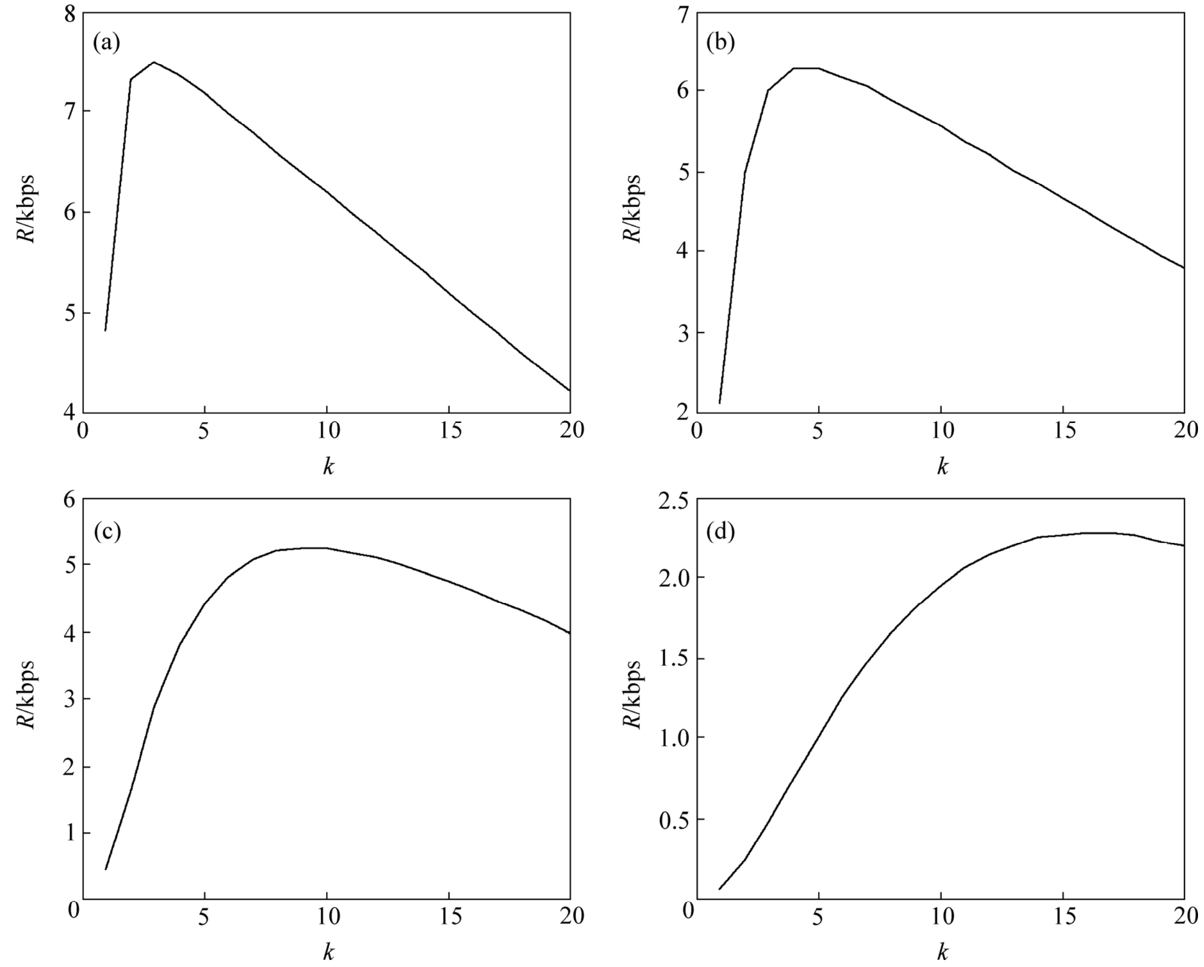

Fig. 7 Throughput versus P_U=0.8(a), throughput versus P_U=0.6(b), throughput versus P_U=0.4(c) and throughput versus P_U=0.2(d)

Table 1 Maximal throughput of each channel and average throughput of each user

P_U	Maximal throughput/kbps	Optimal number of users	Average throughput/kbps
0.8	7.50	3	2.50
0.6	6.28	5	1.26
0.4	5.24	9	2.59
0.2	2.28	16	0.14

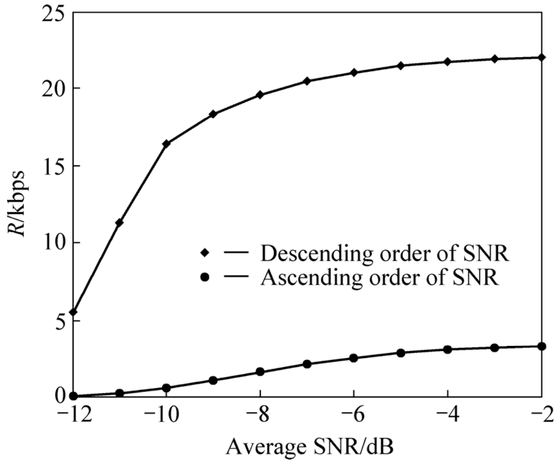

Fig. 8 Throughput versus different orders of SNR

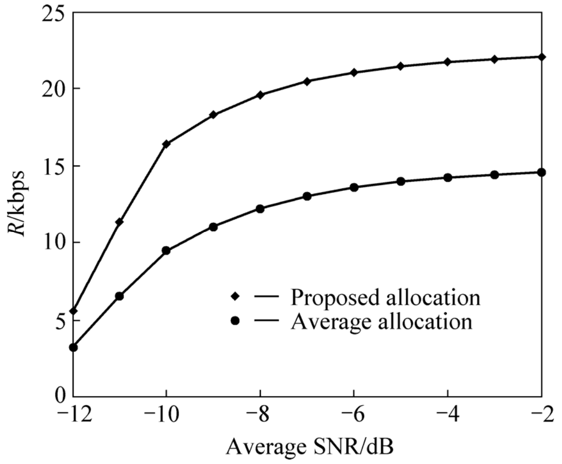

Fig. 9 Throughput comparison between different allocation algorithms

6 Conclusions

The frame of cooperative spectrum sensing is divided into multiple transmission slots and one sensing slot including local energy detection and cooperative overhead. A joint optimization problem of sensing time and number of cooperative users is proposed to maximize the throughput of the CR network, subject to the constraints of false alarm probability and detection probability. The allocation algorithm of CR uses is proposed to preferentially allocate the users to the channel with high utilization probability. The simulation results show that the significant improvement on the throughput can be achieved through the proposed algorithm. The optimization of AND-rule and K-OUT-N-rule cooperative spectrum sensing will be taken into account in the future work.

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