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Application of response surface method for contact fatigue reliability analysis of spur gear with consideration of EHL

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Abstract: In order to consider the effects of elastohydrodynamic lubrication (EHL) on contact fatigue reliability of spur gear, an accurate and efficient method that combines with response surface method (RSM) and first order second moment method (FOSM) was developed for estimating the contact fatigue reliability of spur gear under EHL. The mechanical model of contact stress analysis of spur gear under EHL was established, in which the oil film pressure was mapped into hertz contact zone. Considering the randomness of EHL, material properties and fatigue strength correction factors, the proposed method was used to analyze the contact fatigue reliability of spur gear under EHL. Compared with the results of 1.5×10^5 by traditional Monte-Carlo, the difference between the two failure probability results calculated by the above mentioned methods is 2.2×10^{-4}, the relative error of the failure probability results is 26.8%, and time-consuming only accounts for 0.14% of the traditional Monte-Carlo method (MCM). Sensitivity analysis results are in very good agreement with practical cognition. Analysis results show that the proposed method is precise and efficient, and could correctly reflect the influence of EHL on contact fatigue reliability of spur gear.

Key words: response surface; contact fatigue; reliability; spur gear; elastohydrodynamic lubrication

1 Introduction

Considering the randomness of load, manufacture and material properties, probabilistic design is used instead of deterministic design in contact fatigue reliability analysis of gear. PENG et al [1] used a stochastic finite element method to analyze bending fatigue reliability of gear, in which the randomness of geometric parameters, material properties and load was considered. ZHANG et al [2] proposed a perturbation approach to investigate the contact fatigue reliability of gear, in which the coefficients of analytical formula of contact stress and contact fatigue strength were defined as random variables. CHEN and SUN [3] proposed a model of multi-source data fusion to perform contact fatigue reliability of gear, in which the randomness of geometric parameters and operating conditions was considered. ALEMAYEHU and EKĜWAKO-OSIR [4] presented a novel probabilistic multi-body dynamic analysis approach to estimate bending fatigue reliability of spur gear, in which the randomness of geometric parameters, material properties and operating conditions was considered.

Actually, for the high-speed running gear, it would form elastohydrodynamic lubrication (EHL) between two contact surfaces. DAWSON [5] and the lubrication research group of the ASME [6] studied the metallic contact fatigue under oil lubrication conditions, respectively. They both proposed that the contact fatigue life has an exponential relationship with the oil film thickness of EHL. Recently, the research results of EHL also showed that due to the effect of oil film pressure, the two contact surfaces were separated by oil film, the friction conditions of gear contact surface were improved, the growth of initial crack on the subsurface were relieved and the contact fatigue life of gear was prolonged [7−9]. However, EHL was not considered in the above mentioned research works [1−4]. Thus, in the precise calculation of gear contact fatigue reliability, taking into account of EHL is of extreme necessity.

For the reasons that there are a large number of random variables in calculation of contact fatigue reliability of gear, the EHL equations are highly nonlinear, and limit state function on the basis of stress−strength interference theory is hard to express in explicit formula. Although using MCM [10] and stochastic finite element method [1, 11] was proved to be very effective

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to this kind of problem, it involved much computational effort which researchers sometimes cannot afford. Although using FOSM [12] and perturbation method [2, 13] was proved to be efficient, its application was limited to not very complicated explicit limit state function. The basic idea of response surface method is to approximate the relation between input random variables and output random variables by a simple response surface function. Then, the established limit state function is used for reliability analysis instead of finite element model. This approximate method is effective for complicated and nonlinear structure reliability analysis [14]. Thereby, with the consideration of the randomness of EHL, material properties and fatigue strength correction factors, an efficient and accurate reliability analysis approach that combines with RSM and FOSM is developed for contact fatigue reliability analysis of spur gear.

2 EHL model of spur gear

Suppose that lubricant is Newtonian fluid. Isothermal line contact elastohydrodynamic lubrication model is adopted for this problem. A coordinate system, where x-axis is the coordinate in direction of motion and y-axis is the coordinate in direction of film thickness, is established. EHL equations are as follows [15−16].

2.1 Reynolds equation

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\rho h^3}{\eta}\frac{\mathrm{d}p}{\mathrm{d}x}\right)=12u\frac{\mathrm{d}(\rho h)}{\mathrm{d}x} \tag{1}$$

where ρ and η are the lubricant density and viscosity, respectively; u is the average surface velocity in the x-direction; p and h are the oil film pressure and thickness, respectively.

Boundary conditions of Reynolds equation:

$$\begin{cases} x=x_0,\ p=0\ \text{(entrance region)} \\ x=x_\mathrm{e},\ p=\left.\dfrac{\partial p}{\partial x}\right|_{x=x_\mathrm{e}}=0\ \text{(exit region)} \end{cases}$$

2.2 Elasticity equation

Elasticity equation contains initial gap, central film thickness and elasticity deformation:

$$h(x)=h_0+\frac{x^2}{2R}-\frac{2}{\pi E'}\int_{x_0}^{x_\mathrm{e}}p(s)\ln(s-x)^2\,\mathrm{d}s \tag{2}$$

where h_0 is a constant; R is the equivalent contact radius; E' is the equivalent elastic modulus; s is the distance between an arbitrary loading distribution $p(s)$ and coordinate original.

2.3 Pressure−viscosity equation

Reynolds model is adopted in pressure−viscosity relation.

$$\eta=\eta_0\exp\{(\ln\eta_0+9.67)[(1+\frac{p}{p_0})^z-1]\} \tag{3}$$

where η_0 is the lubricant viscosity at ambient pressure; p_0 is the pressure coefficient; z is the mineral oil coefficient.

2.4 Pressure−density equation

Pressure−density equation proposed by DOWSON and HIGGINSON is adopted.

$$\rho=\rho_0\left(1+\frac{0.6p}{1+1.7p}\right) \tag{4}$$

where ρ_0 is the lubricant density at ambient pressure.

2.5 Load equation

$$w=\int_{-\infty}^{x}p\,\mathrm{d}x \tag{5}$$

where w is the external load per unit cylinder.

Seven dimensionless parameters $X=x/b$, $H=hR/b^2$, $\rho^*=\rho/\rho_0$, $\eta^*=\eta/\eta_0$, $P=p/p_\mathrm{H}$, $W=w/(E'R)$ and $U^*=\eta_0 u/(E'R)$ are introduced to reduce unknowns of equations, where b is hertz half width and p_H is maximum hertz pressure. Numerical analysis results are obtained by substituting these dimensionless parameters into EHL equations [16].

3 Response surface method

Compared with higher order polynomial, quadratic polynomial is widely used in RSM because there have less coefficients to be adjusted. BAI et al [17] used quadratic polynomial without the intercrossing terms to approximate real limit state function. In order to achieve higher calculation accuracy, quadratic polynomial with intercrossing terms is adopted as

$$y=C_0+\sum_{i=1}^{r}C_i x_i+\sum_{i=1}^{r}\sum_{j=1}^{r}C_{ij}x_i x_j \tag{6}$$

where C_0, C_i and C_{ij} are the regression coefficients, and undetermined coefficients are $n(n-1)/2+2n+1$ in total; $\boldsymbol{X}=(x_1, x_2, \cdots, x_r)$ is a random variables vector; $\boldsymbol{Y}=(y_1, y_2, \cdots, y_n)$ is a response values vector.

The accuracy of response surface fitting heavily depends on sampling technology. In a word, the selecting datasets must be representative over the whole design space. Thus, the central composite design method (CCD) is adopted. Sample value at an arbitrarily test value p_i is calculated by

$$\int_{-\infty}^{x_p}f(x)\mathrm{d}x=p_i,\ i=1,2,3,4,5 \tag{7}$$

where $f(x)$ is the probability density function (PDF) of random variables; p_i is the test level. In this work, p_1=0.005, p_2=0.05, p_3=0.5, p_4=0.95, p_5=0.995.

Since x_j and x_i both can be calculated by Eq. (7), the intercrossing terms and the square terms of Eq. (6) can be eliminated by a new variable x_{r+1} (if x_1x_1 is replaced by x_{r+1}, and so is the rest x_ix_j), then Eq. (6) is translated from a quadratic polynomial equation into a multivariate linear regression equation.

$$y = C_0 + \sum_{i=1}^{n(n+3)/2} C_i x_i \tag{8}$$

In order to obtain the response surface, the least square method is used to estimate the regression coefficients. Because $n(n+3)/2$ regression terms are contained in Eq. (8), in order to make the regression model as simple as possible and keep the regression accuracy in the mean time, forward stepwise regression method is adopted.

4 Performance evaluation of response surface model

Though the response surface model can be used for reliability analysis and reliability sensitivity analysis, the significance of the response surface model is still needed to evaluate via F test.

Choosing statistical variables

$$F = \frac{S_{\mathrm{R}}/k}{S_{\mathrm{E}}/(m-k-1)} \sim F(k, m-k-1) \tag{9}$$

where S_{R} is the regression sum of squares; S_{E} is the residual sum of squares; k is the degrees of freedom of S_{R}; m is the number of samples.

If $F>F_\alpha(k, m-k-1)$ at the given confidence level α, suggesting the significance of regression effect; otherwise, suggesting the insignificance of regression effect.

5 Reliability and reliability sensitivity analysis

5.1 First-order second-moment method

A first order Taylor series expansion of the established limit state equation y is obtained at the mean value $u_X = (u_{x_1}, u_{x_2}, \cdots, u_{x_r})$ of basic variables.

$$y(x_1, x_2, \cdots, x_n) \approx y(u_{x_1}, u_{x_2}, \cdots, u_{x_r}) + \sum_{i=1}^{r} \left(\frac{\partial y}{\partial x_i}\right)_{u_X} (x_i - u_{x_i}) \tag{10}$$

Suppose that all basic variables are independent respectively, the mean value u and variance σ^2 of limit state function can be obtained as

$$\begin{cases} u = y(u_{x_1}, u_{x_2}, \cdots u_{x_r}) \\ \sigma^2 = \sum_{i=1}^{r} \left(\frac{\partial y}{\partial x_i}\right)_{u_X}^2 \sigma_{x_i}^2 \end{cases} \tag{11}$$

Reliability index is defined as

$$\beta = u/\sigma \tag{12}$$

Since linear combination of normal distribution still follows normal distribution, reliability of limit state function can be calculated as

$$P_{\mathrm{r}} = P\{y>0\} = P\{\frac{y-u}{\sigma} > -\frac{u}{\sigma}\} = \Phi(\beta) \tag{13}$$

5.2 Sensitivity analysis

When the reliability sensitivity is defined as the partial derivation of the failure probability P_{f} with respect to the mean value and variance of the random variables, according to the derivative rule for compound function, reliability sensitivity calculation formula can be deduced as

$$\begin{cases} \dfrac{\partial P_{\mathrm{f}}}{\partial u_{x_i}} = \dfrac{\partial P_{\mathrm{f}}}{\partial \beta}\dfrac{\partial \beta}{\partial u_{x_i}} \\ \dfrac{\partial P_{\mathrm{f}}}{\partial \sigma_{x_i}} = \dfrac{\partial P_{\mathrm{f}}}{\partial \beta}\dfrac{\partial \beta}{\partial \sigma_{x_i}} \end{cases} \tag{14}$$

where

$$\begin{cases} P_{\mathrm{f}} = 1 - P_{\mathrm{r}} \\ \dfrac{\partial P_{\mathrm{f}}}{\partial \beta} = -\varphi(\beta) \\ \dfrac{\partial \beta}{\partial u_{x_i}} = \dfrac{\left(\frac{\partial y}{\partial x_i}\right)_{u_{x_i}}}{\sigma} \\ \dfrac{\partial \beta}{\partial \sigma_{x_i}} = \dfrac{\left(\frac{\partial y}{\partial x_i}\right)_{u_{x_i}}^2 \sigma_{x_i} u}{\sigma^3} \end{cases} \tag{15}$$

6 Numerical example

6.1 Contact stress analysis of spur gear with consideration of EHL

Taking a pair of spur gear of main helicopter reduction as an example, the contact fatigue reliability analysis of pinion is investigated by the proposed method, and the results are compared with that resulted from the traditional MCM.

Basic parameters and operating conditions of gear pair are given in Table 1.

By analyzing the change rule of gear meshing process, five special positions(seven meshing points) are used to represent the whole gear meshing process: the coming into contact point A; double teeth into single tooth alternating point B (further analysis, this point can be represented by a transient point of double teeth meshing B_- and a transient point of single tooth meshing B_+); pitch point J; single tooth into double teeth

Table 1 Parameters and operating conditions of gear-pair

Item	Pin	Gear
Number of teeth	32	103
Module/mm	3.5	3.5
Gear width/mm	45	45
Pressure angle/(°)	22.5	22.5
Rotational velocity/(r·min^{-1})	7500	—
Power/kW	600	—
Kinetic energy/(Pa·s)	0.014	
Material name	20Cr2Ni4A	
Tensile strength/MPa	1175	
Modulous/MPa	2.07×10^5	
Poisson ratio	0.29	

alternating point C (similarly with the point B, this point can be represented by a transient point of single tooth meshing C_- and a transient point of double teeth meshing C_+); the coming out of point D, then the EHL numerical solution of the above typical meshing points are used to represent the EHL characteristics of the whole meshing process for gear, depicted in Fig. 1.

Distribution of oil film pressure along gear width at the B_+ point is depicted in Fig. 2, and analysis result figures of other points are omitted.

Assume that the oil film stability exists without being broken. Considering the oil film pressure in hertz

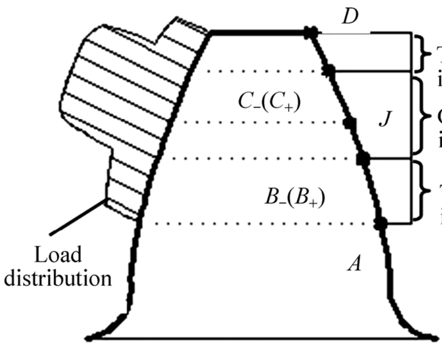

Fig. 1 Load distribution along meshing line

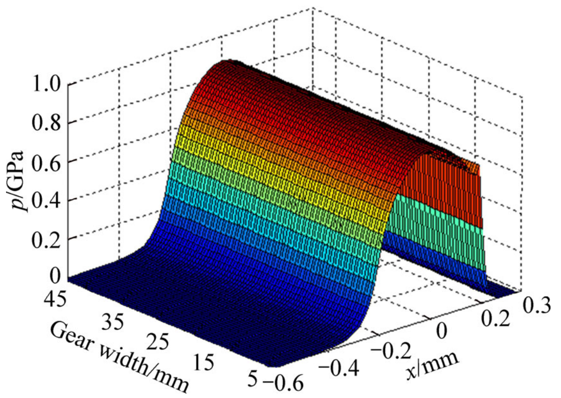

Fig. 2 Distribution of oil film pressure along gear width on B_+ point

contact deformation zone is irregular (as shown in Fig. 2), to maximally embody the effect of the oil film pressure on gear contact surface, the oil film pressure is mapped into hertz contact zone [18−20].

Take point B_+ as an example (so is the rest points). Because oil film pressure is a solution within hertz contact deformation zone, loading area which is limited by hertz contact deformation zone is equally divided into five narrow and long loading surfaces along the hertz contact width (as shown in Fig. 3).

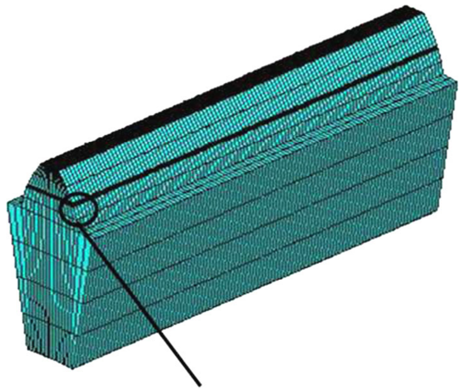

Fig. 3 Division of loading surface

Oil film pressure within hertz contact zone of the point B_+ is also equally divided into five parts. For each part, the oil film pressure is averaged. Then, the average oil film pressures are loaded on the above corresponding loading surfaces. Mesh generation for three-dimensional model of gear is performed by using 20-node solid 186 isoparametric elements and refining the area of hertz contact deformation zone, then 21186 units and 95566 nodes are included in the finial finite element model (see Fig. 3). Through the finite element analysis, contact stress of meshing point B_+ under the effect of oil film pressure is obtained.

In order to investigate the change rules of contact stress during the whole meshing process of gear, contact stress analysis of other points is carried out in the same way as point B_+. Contact stress analysis results are shown in Fig. 4, where roll angle is the track of driving gear from the beginning point to the meshing point along meshing line.

6.2 Establishing response surface model

According to Fig. 4, the maximum contact stress of gear takes place in point B_+, which is most likely to pitting. Therefore, the contact mechanical model of point B_+ is adopted to analyze the contact fatigue reliability of spur gear.

6.2.1 Random variables selection and distribution parameter determination

Considering the randomness of EHL, material

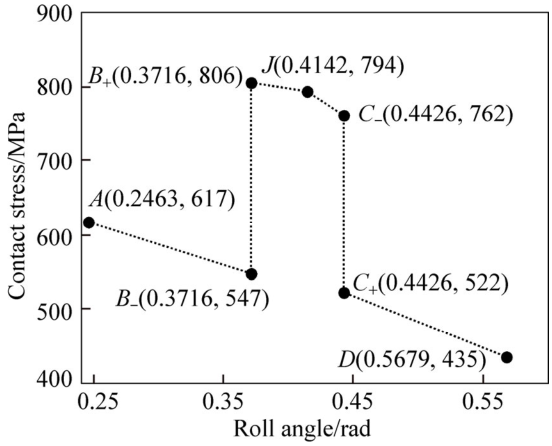

Fig. 4 Contact stress along meshing line

properties and the gear contact fatigue strength, random variables are finally determined as follows: initial viscosity of lubricant η, rotational velocity n, elastic modulus E, Poisson ratio v, material density ρ_m, effective stress concentration coefficient k_t, surface quality coefficient β, size coefficient ε and material contact fatigue limit σ_{-1}. According to the statistics from Ref. [21], and combining with the specific conditions of this work, PDF types and probabilistic parameters of each variable are determined as listed in Table 2.

Table 2 Material, usage, strength modify factor variables

Variable	PDF	Mean	COV
E/Pa	Normal	2.07×10^{11}	0.03
V	Normal	0.29	0.01
η/(Pa·s)	Normal	0.014	0.01
n/(r·min^{-1})	Normal	7500	0.03
ρ_m/(kg·m^{-3})	Normal	7900	0.05
k_t	Normal	1.3	0.01
β	Normal	1	0.01
ε	Normal	0.76	0.06
σ_{-1}/Pa	Normal	1.356×10^9	0.03

6.2.2 Response surface fitting of oil film pressure

Oil film pressure is the external load of contact stress analysis and the response surface function of E, v, η and n. If the effects of these random variables on oil film pressure are directly embodied in the contact fatigue reliability analysis model of spur gear, the feasibility is worth considering. Fortunately, RSM could well solve that problem.

The central composite method is used to arrange experimental scheme for the above four variables. Sample values are calculated by Eq. (7). Combining with EHL equations, 25 deterministic solutions in total are completed. Supposing that the five average oil film pressures from entrance to exit are expressed by P_{sf1}, P_{sf2}, P_{sf3}, P_{sf4} and P_{sf5}, respectively, the sample values of the five average oil film pressures are depicted in Fig. 5. The response surface equations of the five average oil film pressures are finally fitted as Eq. (16) according to Eq. (8).

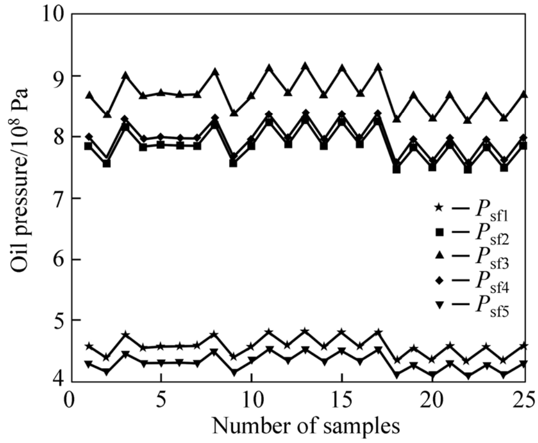

Fig. 5 Samples of oil film pressure

$$\begin{cases} P_{sf1}=4.5821\times10^8+2.0790\times10^{-3}E+1.3871\times10^8 v- \\ \quad 3.6129\times10^8\eta-6.6501\times10^4 n-1.0833\times10^{-15}E^2+ \\ \quad 3.3126n^2-7.0531\times10^{-8}E\cdot n \\ P_{sf2}=7.8008\times10^8+3.6031\times10^{-3}E+2.9531\times10^7 v- \\ \quad 6.0197\times10^8\eta-1.0403\times10^5 n-2.4466\times \\ \quad 10^{-15}E^2+5.2358n^2+1.0197\times10^{-3}E\cdot v-1.3609\times \\ \quad 10^{-7}E\cdot n \\ P_{sf3}=8.1565\times10^8+4.1191\times10^{-3}E+8.7075\times10^7 v- \\ \quad 3.0407\times10^8\eta-1.1115\times10^5 n-2.7877\times10^{-15}E^2+ \\ \quad 5.6035n^2+8.8770\times10^{-4}E\cdot v-1.5255\times10^{-7}E\cdot n \\ P_{sf4}=6.3302\times10^8+4.2020\times10^{-3}E+2.5611\times10^8 v+ \\ \quad 3.1389\times10^8\eta-9.3640\times10^4 n-2.7684\times10^{-15}E^2+ \\ \quad 4.7534n^2-1.4879\times10^{-7}E\cdot n \\ P_{sf5}=4.7250\times10^8+2.0037\times10^{-3}E+1.1426\times10^8 v- \\ \quad 1.7609\times10^9\eta-6.31730\times10^4 n-1.2423\times10^{-15}E^2+ \\ \quad 3.1421n^2-7.4419\times10^{-8}E\cdot n \end{cases} \tag{16}$$

According to Eq. (9), in the case of confidence level α=0.05, the significance test of the response surface function of oil film pressure is listed in Table 3. Results show that the regression effects of the response surface function of oil film pressure are of extreme significance.

6.2.3 Response surface fitting of limit state function

According to stress−strength interference theory, limit state function is defined as

$$y=\sigma'_{-1}-\sigma_j \tag{17}$$

where σ_j is the gear contact stress and σ'_{-1} is the gear

Table 3 *F* test result of oil film pressure regression equation

Test	ρ_{sf1}	ρ_{sf2}	ρ_{sf3}	ρ_{sf4}	ρ_{sf5}
F	6.6700×10^4	2.6269×10^5	3.8856×10^5	3.0810×10^5	3.1038×10^4
$F_\alpha\ (k, m-k-1)$	$F_{0.05}(7, 17)$	$F_{0.05}(8, 16)$	$F_{0.05}(8, 16)$	$F_{0.05}(7, 17)$	$F_{0.05}(7, 17)$
	2.6143	2.5911	2.5911	2.6143	2.6143
Test result	Significance	Significance	Significance	Significance	Significance

contact fatigue strength limit.

$$\sigma'_{-1}=\frac{\varepsilon\beta}{k_t}\sigma_{-1} \tag{18}$$

The central composite method is used to arrange experimental scheme for all variables, η, n, E, v, ρ_m, k_t, β, ε and σ_{-1}. Sample values are calculated by Eq. (7). By calling the deterministic finite element analysis of 147 times, the values of y are obtained as shown in Fig. 6. The response surface equations of limit state function y is finally fitted according to Eq. (8), and listed in Table 4.

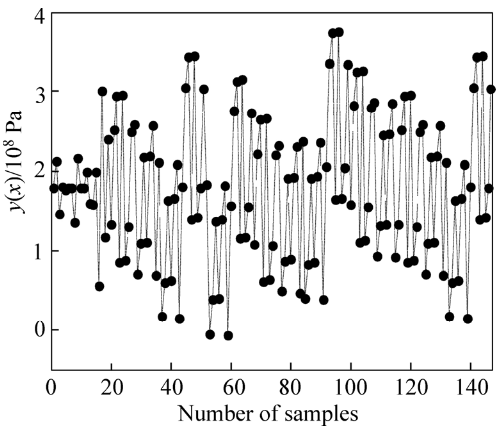

Fig. 6 Samples of limit state function

According to Eq. (9), in the case of confidence level α=0.05, the significance test of the response surface function of limit state function is listed in Table 5. Results show that the regression effect of the response surface function of limit state function is of extreme significance.

6.3 Reliability and reliability sensitivity analysis

6.3.1 Reliability results and analysis

Parameter μ, σ, β and P_r are obtained from Eqs. (11) to (13). In order to validate the accuracy and efficiency of the proposed approach, 1.5×10^5 numerical simulations using traditional MCM are performed on Shuguang 5000 high performance computing platform. According to the comparison results listed in Table 6, the difference between the two failure probability results P_f is 2.2×10^{-4}, the relative error R_e is 26.8%, and time-consuming only accounts for 0.14% of the traditional MCM, which shows that the proposed approach is precise and efficient.

Table 4 Limit state function regression equation

Regression variable	Coefficient
Constant	-5.8701×10^8
n	1.2843×10^5
η	2.5643×10^8
k_t	-6.7748×10^5
E	-4.8939×10^{-3}
v	-2.0161×10^8
β	-8.5100×10^3
ε	-1.1650×10^4
σ_{-1}	-7.8729×10^{-6}
k_t^2	4.6946×10^8
E^2	1.1844×10^{-15}
n^2	-8.8094
$E\cdot v$	-6.7407×10^{-4}
$E\cdot n$	3.3229×10^{-7}
$v\cdot n$	1.7645×10^4
$k_t\cdot\beta$	-6.0996×10^8
$k_t\cdot\varepsilon$	-8.0258×10^8
$k_t\cdot\sigma_{-1}$	-4.4983×10^{-1}
$\beta\cdot\varepsilon$	1.0434×10^9
$\beta\cdot\sigma_{-1}$	5.8478×10^{-1}
$\varepsilon\cdot\sigma_{-1}$	7.6944×10^{-1}

Table 5 *F* test result of limit state regression equation

Test	Value
F	5.25686×10^6
$F_\alpha(k, m-k-1)$	$F_{0.05}(20,100)$
	1.6764
Test result	Significance

Table 6 Reliability results for RSM/FORM and MCM

Method	μ	σ	β	P_f	R_e/%
MCM	1.7814×10^8	5.8010×10^7	3.0708	8.2×10^{-4}	26.8
RSM/FORM	1.7849×10^8	5.7983×10^7	3.0783	1.04×10^{-3}	

Number of runs	Computing speed of Shuguang 5000/(trillion·s^{-1})	Time-consuming/h
150000	10	421
147		0.62

6.3.2 Sensitivity results and analysis

Sensitivity vector $\partial P_f/\partial u_X$, the partial derivation of the failure probability P_f with respect to the mean value of the random variables, is obtained from Eq. (14) as listed in Table 7. According to the sensitivity vector $\partial P_f/\partial u_X$, conclusions are obtained: 1) The reliability increases as n, η, β, ε and σ_{-1} increase; 2) The reliability decreases as k_t, E and v increase; 3) The ρ_m has no effect on reliability and can be treated as deterministic variable. The above conclusions are in very good agreement with results in Ref. [15], namely, the increments of n and η could help the oil film to form and improve the friction conditions of gear contact surface, therefore, the gear contact fatigue life is prolonged; the increments of E and v could lead to a higher equivalent elastic modulus E', which will make the oil film thickness become thinner and shorten the gear contact fatigue life. In additional, according to Eq. (18), the increments of β, ε and σ_{-1} can improve the contact fatigue strength limit of gear σ'_{-1}; while the increments of k_t can reduce the contact fatigue limit of gear σ'_{-1}. On the basis of absolute value of sensitivity vector $\partial P_r/\partial u_X$, the order is $u_\varepsilon > u_\beta > u_{k_t} > u_\eta > u_v > u_n > u_{\sigma_{-1}} > u_E > u_{\rho_m}$. In other words, the reliability is very sensitive to ε, no sensitive to ρ_m, and moderately sensitive to the rest.

Table 7 Sensitivity analysis

Variable	$\partial P_f/\partial u_X$	Variable	$\partial P_f/\partial \sigma_X$
$\partial P_f/\partial u_n$	-4.0933×10^{-6}	$\partial P_f/\partial u_n$	3.3793×10^{-6}
$\partial P_f/\partial u_{k_i}$	3.6734×10^{-2}	$\partial P_f/\partial u_{k_i}$	1.5464×10^{-2}
$\partial P_f/\partial u_\eta$	-1.5443×10^{-2}	$\partial P_f/\partial u_\eta$	2.9013×10^{-5}
$\partial P_f/\partial u_E$	1.2436×10^{-13}	$\partial P_f/\partial u_E$	8.4664×10^{-14}
$\partial P_f/\partial u_v$	1.2441×10^{-2}	$\partial P_f/\partial u_v$	3.9569×10^{-4}
$\partial P_f/\partial u_\beta$	-4.7755×10^{-2}	$\partial P_f/\partial u_\beta$	2.0105×10^{-2}
$\partial P_f/\partial u_\varepsilon$	-6.2835×10^{-2}	$\partial P_f/\partial u_\varepsilon$	1.5872×10^{-1}
$\partial P_f/\partial u_{\sigma_{-1}}$	-3.5216×10^{-11}	$\partial P_f/\partial u_{\sigma_{-1}}$	4.4475×10^{-11}
$\partial P_f/\partial u_{\rho_m}$	0	$\partial P_f/\partial u_{\rho_m}$	0

Sensitivity vector $\partial P_f/\partial u_X$, the partial derivation of the failure probability P_f with respect to variance of the random variables, is obtained from Eq. (14), listed in Table 7. According to the sensitivity vector $\partial P_f/\partial u_X$, the reliability decrease as the variance of all the basic random variables increase. Thus, those distributed parameters should be strictly controlled during the course of design and manufacture.

7 Conclusions and future work

1) EHL equations are approximated by a quadratic polynomial with intercrossing terms, and mapped into hertz contact zone. Then, the EHL effects on contact fatigue reliability of spur gear are successfully considered in the proposed approach.

2) Compared with the traditional MCM, the proposed approach is accurate and efficient.

3) Sensitive analysis results are in very good agreement with actual cognition, which proves that the proposed approach could correctly reflect the influence of EHL on contact fatigue reliability of spur gear, and is able to enhance the calculation accuracy of the contact fatigue reliability of spur gear.

4) On the basis of sensitive analysis results, it will help designers to critically consider certain parameters that are more significant for improving or hampering contact fatigue reliability of spur gear.

5) The future work of the problem is to consider the influence of temperature on the proposed model.

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